ALGEMEEN 1 / GENERAL 1 Gr 7 - 9

GETALLE / NUMBERS

1. Die optellingsinverse van
3 is –3; -½ is ½; 1,0075 is –1,0075
Die som van 'n getal en sy optellings-inverse is altyd gelyk aan 0.

The additive inverse of
The sum of a number and its additive inverse is always equal to 0.

2. Die resiprook (vermenigvuldigingsinverse) van
The reciprocal (multiplicative inverse) of
3 is 1/3; -½ is –2; 75 is $\frac{1}{75}$
Die produk van 'n getal en sy resiprook is altyd gelyk aan 1.

The product of a number and its reciprocal is always equal to 1.

3.1A {Priemgetalle} = {2; 3; 5; 7; 11; 13; 17; 19; 23; 29; . . .}

In 'n *versameling* getalle word geen getal (element) herhaal nie.
Die krulhakies {. . . } word gebruik om *versamelings* aan te dui.

In 'n lys getalle kan die getalle herhaal word.

Die getal 1 is nie 'n priemgetal nie.
Die enigste ewe priemgetal is 2.
'n Heelgetal, p, is priem as $p > 1$ en slegs deelbaar is deur 1 en homself.

'n Natuurlike getal, p, is 'n *saamgestelde* getal as $p > 1$ en nie priem is nie.

3.1E {Prime numbers} = {2; 3; 5; 7; 11; 13; 17; 19; 23; 29; . . .}

In a *set* of numbers no number (element) is repeated.
The curly braces {. . . } are used to indicate *sets*.

In a list of numbers the numbers can be repeated.

The number 1 is not a prime number.
The only even prime number is 2.
An integer, p, is prime if $p > 1$ and if it is only divisible by 1 and itself.

A natural number, p, is a *composite* number if $p > 1$ and not prime.

3.2A Onderling priem; relatief priem

Getalle wat relatief priem is, is nie noodwendig priemgetalle nie, maar kan nie verder vereenvoudig word ten opsigte van mekaar nie.

Byvoorbeeld, breuke: die teller en die noemer kan nie verder vereenvoudig word nie; die GGD (teller, noemer) = 1. Dus is die teller en die noemer relatief priem.

6 en 7, 16 en 33, 30 en 49 is voorbeelde hiervan; $\frac{6}{7}$, $\frac{16}{33}$ en $\frac{30}{49}$ is breuke in eenvoudigste vorm.

6 en 15 is nie priemgetalle nie en hulle is ook nie relatief priem nie, want hulle het 'n gemene faktor (deler), naamlik 3.
3 en 15 is nie relatief priem nie, al is 3 'n priemgetal, want hulle het 'n gemene faktor (deler), 3.

3.2E Mutually prime; co-prime; relative prime

Numbers that are relative prime are not necessarily prime numbers, but cannot be simplified further with regard to each another.

For example, fractions: the numerator and the denominator cannot be simplified (reduced) further; the GCD(numerator, denominator) = 1. Therefore the numerator and denominator are relative prime.

6 and 7, 16 and 33, 30 and 49 are examples; $\frac{6}{7}$, $\frac{16}{33}$ and $\frac{30}{49}$ are fractions in simplest form.

6 and 15 are not prime numbers and not relative prime, since they have a common factor (divisor) which is 3.
3 and 15 are also not relatively prime, although 3 is a prime number, since they have a common factor 3.

3.3A Priemgetalpare

'n Priemgetal het slegs twee verskillende positiewe heelgetalfaktore, naamlik 1 en die getal self.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, . . .

Nuttig om te ken: $7 \times 11 \times 13 = 1001$ en $7 \times 11 \times 13 \times 37 = 10101$

Priemgetalpare: priemgetalle wat met 2 verskil

3 ; 5 → 5 – 3 = 2 of |3 – 5| =2

5 ; 7 → 7 – 5 = 2 of |5 – 7| = 2

11 ; 13 → 13 – 11 = 2 of |11- 13| = 2

17 ; 19 → 19 – 17 = 2 of |17 – 19| = 2

29 ; 31 → 31 – 29 = 2 of |29 – 31| = 2

Skryf nog sulke pare neer.

Nog interessante priemgetalpatrone:

7 ; 11	13 ; 17	19 ; 23	37 ; 41		
13 ; 31	17 ; 71	37 ; 73	79 ; 97	107 ; 701	113 ; 311
149 ; 941	157 ; 751				

3.3E Twin primes

A prime number has only two distinct positive integer factors, i.e. 1 and the number itself.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, . . .

Useful to know: $7 \times 11 \times 13 = 1001$ and $7 \times 11 \times 13 \times 37 = 10101$

Twin primes: Prime numbers that differ by 2

3 ; 5 → 5 – 3 = 2 of |3 – 5| =2

5 ; 7 → 7 – 5 = 2 of |5 – 7| = 2

11 ; 13 → 13 – 11 = 2 of |11- 13| = 2

17 ; 19 → 19 – 17 = 2 of |17 – 19| = 2

29 ; 31 → 31 – 29 = 2 of |29 – 31| = 2

Write down more such pairs.

More interesting prime number patterns:

7 ; 11 13 ; 17 19 ; 23 37 ; 41

13 ; 31 17 ; 71 37 ; 73 79 ; 97 107 ; 701 113 ; 311
149 ; 941 157 ; 751

3.4A Priemgetaldrietalle: $3^{de} - 1^{ste} = 6$.

(5, 7, 11) → 11 – 5 = 6; 7 – 5 = 2; 11 – 7 = 4
(7, 11, 13) → 13 – 7 = 6; 11 – 7 = 4; 13 – 11 = 2
(11, 13, 17) → 17 – 11 = 6; 13 – 11 = 2; 17 – 4 = 13
. . .
(97, 101, 103) → 103 – 97 = 6; 101 – 97 = 4; 103 – 101 = 2
. . .

3.4E Prime number triplets: $3^{rd} - 1^{st} = 6$.

(5, 7, 11) → 11 – 5 = 6; 7 – 5 = 2; 11 – 7 = 4
(7, 11, 13) → 13 – 7 = 6; 11 – 7 = 4; 13 – 11 = 2
(11, 13, 17) → 17 – 11 = 6; 13 – 11 = 2; 17 – 4 = 13
. . .
(97, 101, 103) → 103 – 97 = 6; 101 – 97 = 4; 103 – 101 = 2
. . .

4. Jaartalle / Year numbers

- 2017 = $44^2 + 9^2$
- (1855, 792, 2017) is a Pythagorean triple because $2017^2 = 1855^2 + 792^2$.
- 2027 en 2029 is ook priemgetalle.
- 2017 = $7^3 + 7^3 + 11^3$.

- 2018 is nie 'n priemgetal nie.
- 2018 = 2 × 1009 en 1009 is 'n priemgetal.
- Som van 2018 se delers = 1 + 2 + 1009 + 2018 = 3030.
- $\sqrt{2018} \cong 44.9 < 45$
- Som van 2018 se priemfaktore = 2 + 1009 = 1011 en 1011 = 3 × 337
- 2 + 0 + 1 + 8 = 11 en dus is 2018 deelbaar deur 11.
- 2018^{de} priemgetal = 17551 / 2018^{th} prime number = 17551
- 2018^2 = 4 072 324

- 2019 = 3 × **673** en/and 673 en 3 is albei priemgetalle / are both prime numbers.
- $\sqrt{2019} \cong 44.9 < 45$
- $385^2 + 552^2 = \mathbf{673}^2$
  en / and
- 385 × 3 = 1155, 552 × 3 = 1656, **673** × 3 = 2019
- Therefore, (385; 552; 673) as well as (1155; 1656; 2019) are Pythagorean triplets.

5.A Skrikkeljaar

- 'n Gewone jaar het 365 dae en 'n skrikkeljaar het 366 dae.

- 'n Jaar het 52 vol weke en $52 \times 7 = 364$.
- 'n Mens kan sê dat elke vierde jaar 'n skrikkeljaar is.
- As die jaartal presies deelbaar is deur 4, is dit 'n skrikkeljaar, bv. 1904, 2020, 1808, 2000.
- As die jaartal op twee nulle eindig, moet dit deelbaar wees deur 400 om 'n skrikkeljaar te wees.
  Dus: 1900 is nie 'n skrikkeljaar nie want dit is nie deelbaar deur 400 nie.

5.E Leap year

- A normal year has 365 days and a leap year has 366 days.
- A year has 52 full weeks and we have $52 \times 7 = 364$.
- One can say that every fourth year is a leap year.
- If the year is exactly divisible by 4 it is a leap year, for example 1904, 2020, 1808, 2000.
- If the year ends in two zeros it must be divisible by 400 to be a leap year.
  Therefore: 1900 is not a leap year since it is not divisible by 400.

6. Die getalle **1** en **0** is nie priemgetalle nie en ook nie saamgestelde getalle nie.

The numbers **1** and **0** are not prime numbers and also not composite numbers.

7. {Ewe getalle / Even numbers} = {2; 4; 6; 8; 10; . . .}
Formule / Formula: $2n$, vir/for $n = 1, 2, 3, \ldots$

8. {Onewe getalle / Odd numbers} = {1; 3; 5; 7; 9; 11; . . .}
Formule / Formula: $2n - 1$, vir/for $n = 1, 2, 3, \ldots$
$2n + 1$, vir/for $n = 0, 1, 2, 3, \ldots$

9. {Natuurlike getalle / Natural numbers} = $\mathbb{N}$ = {1; 2; 3; 4; . . .}

10. {Telgetalle / Counting numbers} = $\mathbb{N}_0$ = {0; 1; 2; 3; 4; . . .}

Whole numbers = {0; 1; 2; 3; 4; . . .}

11. {Heelgetalle / Integers} = $\mathbb{Z}$ = {. . . ; -3; -2; -1; 0; 1; 2; 3; . . .}

- {Positiewe heelgetalle / Positive integers} = {1; 2; 3; 4; . . .}
- {Negatiewe heelgetalle / Negative integers} = {. . . ; -3; -2; -1}
- {Nie-positiewe heelgetalle / Non-positive integers} = {. . . ; -3; -2; -1; 0}
- {Nie-negatiewe heelgetalle / Non-negative integers} = {0; 1; 2; 3; 4; . . .}

12.1 $\mathbb{Q}$ = {Rasionale getalle / Rational numbers} = $\{ \frac{a}{b}, b \neq 0, a \in \mathbb{Z}, b \in \mathbb{Z} \}$.

Voorbeelde / Examples: $\frac{22}{7}$; 1,25; $1,\dot{2}\dot{5}$; 10; -6; $-1\frac{3}{8}$; $\sqrt{16}$; $\sqrt[3]{-27}$

In die breuk $\frac{22}{7}$ is die twee getalle albei heelgetalle geskryf as 'n verhouding 22 : 7.

In the fraction $\frac{22}{7}$ the two numbers are both integers written as a ratio 22 : 7.

In bv. $\frac{22}{7}$ is die getal bo die lyn die teller en die getal onder die lyn die noemer.

In $\frac{22}{7}$ the number above the line is the numerator and the number below the line is the denominator.

Alle rasionale getalle kan as breuke geskryf word.

All rational numbers can be written as fractions.

12.2 Repeterende desimale breuke / Recurring decimal fractions

Rasionale getalle kan geskryf word as repeterende desimale breuke.

Rational numbers can be written as recurring decimal fractions.

Die aantal repeterende syfers is altyd
* kleiner as die waarde van die noemer.
* 'n faktor van (getal – 1)

The number of recurring digits is always
* less than the value of the denominator.
* a factor of (number – 1)

Laat die noemer = *n* en die aantal repeterende syfers = *m*.
Dus: *m* is 'n faktor van (*n* – 1) of *m* | (*n*–1).

Let the denominator = *n* and the number of recurring decimals = *m*.
Therefore: *m* is a factor of (*n* – 1) or *m* | (*n*–1).

$\frac{1}{10} = 0{,}1 = 0{,}1\underline{0}000\ldots\ldots\ldots\ldots n = 10,\ m = 1$

$\frac{1}{9} = 0{,}\underline{1}1111\ldots\ldots\ldots\ n = 9,\ m = 1$

$\frac{1}{7} = 0.\underline{142857}142857142857142857142857143\ldots\ldots\ldots\ldots\ n = 7,\ m = 6;\ 6 \mid (7-1)$

$\frac{1}{13} = 0.\underline{076923}076923076923076923076923076923076769\ldots\ldots\ldots\ldots n = 13,\ m = 6;\ 6 \mid (13-1)$

$\frac{1}{17} = 0.\underline{0588235294117647}058823529411764706\ldots\ldots\ldots\ n = 17,\ m = 16$

** Om $0{,}\dot{3}1\dot{2}$ in die vorm *a*/*b* te skryf: / To write $0{,}\dot{3}1\dot{2}$ in the form *a*/*b*:

$$x = 0{,}\dot{3}1\dot{2}$$
$$= 0{,}312312312\ldots$$
$$1000x = 312{,}312312312\ldots$$
$$1000x - x = 312{,}312312312\ldots - 0{,}312312312\ldots$$
$$999x = 312$$
$$x = \frac{312}{999} = \frac{104}{333}$$

** Om $1.2\dot{4}$ in die vorm *a*/*b* te skryf: / To write $1.2\dot{4}$ in the form *a*/*b*:

$$x = 1.2\dot{4}$$
$$= 1{,}24444444\ldots$$
$$10x = 12{,}444444\ldots$$
$$100x = 124{,}444444\ldots$$
$$100x - 10x = 124{,}444444\ldots - 12{,}444444\ldots$$
$$90x = 112$$
$$x = \frac{112}{90} = \frac{56}{45}$$

** $0{,}9 + 0{,}09 + 0{,}009 + \ldots$

is 'n meetkundige reeks. Om elke nuwe term verkry word deur die vorige term met die konstante verhouding (r) te vermenigvuldig. Die eerste term is a.
In hierdie geval is $r = 0{,}09 \div 0{,}9 = 0{,}1$ en $r = 0{,}009 \div 0{,}09 = 0{,}1$.

NB: formule vir die som van 'n oneindige aantal terme van 'n meetkundige reeks is

$$S_\infty = \frac{a}{1-r}\text{, met } a = 0{,}9;\ r = \frac{1}{10}$$

0,9 + 0,09 + 0,009 + . . .
is a geometric series. Each new term is obtained by multiplication of the previous term with the constant ratio (r). The first term is a.
In this case $r = 0{,}09 \div 0{,}9 = 0{,}1$ and $r = 0{,}009 \div 0{,}09 = 0{,}1$.

NB: formula for the sum of an infinite number of terms of a geometric series is

$$S_\infty = \frac{a}{1-r}\text{, with } a = 0{,}9;\ r = \frac{1}{10}$$

Berekening / Calculation

$0{,}\dot{9}$ $= 0{,}9 + 0{,}09 + 0{,}009 + \ldots$ Meetkundige reeks / Geometric series

$$= 0{,}9 + 0{,}9 \times \left(\frac{1}{10}\right)^1 + 0{,}9 \times \left(\frac{1}{10}\right)^2 + \ldots$$

$$= \frac{0{,}9}{1-\frac{1}{10}}$$

$$= \frac{0{,}9}{0{,}9}$$

$= 1$

(**NB:** $\frac{1}{3}+\frac{2}{3}=\frac{3}{3}$; $\frac{1}{3}=0{,}\dot{3}$; $\frac{2}{3}=0{,}\dot{6}$; $\frac{3}{3}=0{,}\dot{9}=1$)

Kortpad / Shortcut:

Noemer: Skryf 'n 9 vir elke repeterende syfer; 'n 0 vir elke syfer wat nie repeteer nie.
Denominator: Write a 9 for each recurring digit; a 0 for each non-recurring digit.

Teller: Skryf al die syfers neer en trek die nie-repeterende syfers af.
Numerator: Write down all the digits and subtract the non-recurring digits.

13. I_r = {Irrasionale getalle / Irrational numbers}

Irrasionale getalle kan glad nie in die vorm

Irrational numbers can never be written in the form:

$$\{\frac{a}{b},\ b \neq 0,\ a \in Z,\ b \in Z\}$$

geskryf word nie.

Irrasionale getalle kan nie as repeterende desimale getalle geskryf word nie.

Irrational numbers cannot be written as recurring decimal numbers.

Voorbeelde van irrasionale getalle / Examples of irrational numbers:
π; 1,131131113. . .; 3,515115111. . .; 0,437437143724373. . .

Die som van twee irrasionale getalle

The sum of two irrational numbers can

kan soms 'n rasionale getal wees: sometimes be a rational number:

$$0{,}121121112... + 3{,}323323332... = 3{,}444444... = 3{,}\dot{4}$$

- 3.14 en $^{22}/_{7}$ is rasionale benaderings vir π.
- 3.14 and $^{22}/_{7}$ are rational approximations for π.

14.A R = {Reële getalle} = $\mathbb{Q} \cup I_r$

Die reële getalle is al die rasionale getalle saam met al die irrasionale getalle. Dit maak nou die getallelyn vol sodat daar nie plek is vir nog getalle nie.

14.E R = {Real numbers} = $\mathbb{Q} \cup I_r$

The real numbers are all the rational number together with all the irrational numbers. Now the number line is filled with numbers and there are no spaces left for more numbers.

X. C = {Komplekse getalle / Complex numbers} . . . . . nie vir nou / not for now

Voorbeelde: Al die reële getalle asook getalle soos

Examples: all the real numbers as well as numbers such as

$$\sqrt{-16}\,;\ \sqrt{-9}+5$$

Dikwels word $\sqrt{-1}$ aangedui met 'n i of 'n j.

$\sqrt{-1}$ is often indicated with an i or a j.

Dus: As $i = \sqrt{-1}$, dan is $i^2 = -1$.

Therefore: If $i = \sqrt{-1}$, then $i^2 = -1$.

$$\sqrt{(-1)^2} = 1$$

$$(\sqrt{-1})^2 = -1 = i^2.$$

$$\sqrt{(-1)^2} \neq (\sqrt{-1})^2$$

Ons kan dus nou die volgende ook faktoriseer:

We can now also factorise the following:

$$a^2 + b^2 = a^2 - b^2i^2 = (a - bi)(a + bi)$$

$i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$, $i^5 = i^1 = i$, $i^6 = i^2 = -1$, $i^7 = i^3 = -i$, $i^8 = i^4 = 1$.

15.A Driehoekgetalle: 1; 3; 6; 10; 15; 21; . . .

Die formule vir 'n driehoekgetal is $\frac{n}{2}(n+1)$, $n = 1, 2, 3, \ldots$ gee 1, 3, 6, 10, …

[of die formule vir 'n driehoekgetal is $\frac{n}{2}(n-1)$, $n = 1, 2, 3, \ldots$ gee 0, 1, 3, 6, 10, …]

Aantal handdrukke = $\frac{n}{2}(n-1)$ waar n die aantal persone in 'n groep is.

Voorbeeld: As daar 6 mense in die vertrek is en elkeen skud hand met elke ander een, dan is daar $\frac{6}{2}(6-1)$ = 15 handdrukke.

In die figuur (onder) kan ons sien dat die driehoek groter word deur elke keer 'n ry onderaan by te sit wat een kolletjie meer het as die vorige ry.

15.E Triangular numbers: 1; 3; 6; 10; 15; 21; . . .

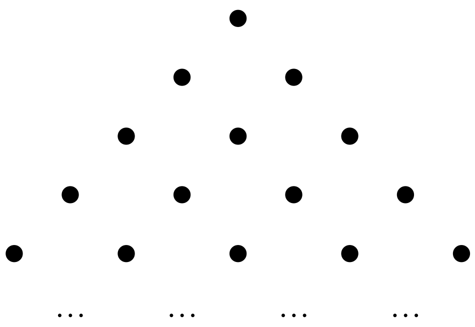
The formula for a triangular number is $\frac{n}{2}(n+1)$, $n = 1, 2, 3, \ldots$ gives 1, 3, 6, 10, …

[or the formula for a triangular number is $\frac{n}{2}(n-1)$, $n = 1, 2, 3, \ldots$ gives 0, 1, 3, 6, 10, …]

Number of handshakes = $\frac{n}{2}(n-1)$ where n is the number of people in a group.

Example: If there are 6 people in a room and each shakes hands with every other person, then there are $\frac{6}{2}(6-1) = 15$ handshakes.

In the figure (below) we can see that the triangle grows by adding a row of dots below that has one dot more than the previous row.



		Totaal Total
ry/row 1	1	1
ry/row 2	1+2=3	3
ry/row 3	3+3=6	6
ry/row 4	6+4=**10**	10
ry/row 5	**10**+5=**15**	**15**
…	…	

16.A Priemfaktore van 'n positiewe heelgetal

Sorg dat jy die deelbaarheidsreëls ken.

Begin:
Hoeveel keer kan jy agtereenvolgens met 2 deel?
Dan deur 3, dan deur 5, dan deur 7, dan deur 11 en so gaan jy aan en werk deur al die priemgetalle totdat jy seker is daar is dat daar nie nog priemgetalle kan indeel nie.

Voorbeelde:

$12 \div 2 = 6$
$6 \div 2 = 3$
$3 \div 3 = 1$
Dus is $12 = 2 \times 2 \times 3$

$364 \div 2 = 182$
$182 \div 2 = 91$
$91 \div 7 = 13$
$13 \div 13 = 1$
Dus is $364 = 2 \times 2 \times 7 \times 13$

$1250 = 125 \times 10 = 25 \times 5 \times 10 = 5 \times 5 \times 5 \times 2 \times 5 = 2 \times 5^4$.
Ons het nou die 1250 dadelik in makliker faktore ontbind en hulle toe verder ontbind totdat al die faktore priemgetalle was.

$3388 = 847 \times 4 = 121 \times 7 \times 4 = 11 \times 11 \times 7 \times 2 \times 2 = 2^2 \times 7 \times 11^2$.

16.E Prime factors of a positive integer

Make sure that you know the divisibility rules.

Start:
How many times can you divide by 2 consecutively?
Then by 3, then by 5, then by 7, then by 11 and continue in this way with the prime numbers until you are sure that no more prime numbers can be used to divide with.

Examples:

$12 \div 2 = 6$
$6 \div 2 = 3$
$3 \div 3 = 1$
Hence, $12 = 2 \times 2 \times 3$

$364 \div 2 = 182$
$182 \div 2 = 91$
$91 \div 7 = 13$
$13 \div 13 = 1$
Hence, $364 = 2 \times 2 \times 7 \times 13$

$1250 = 125 \times 10 = 25 \times 5 \times 10 = 5 \times 5 \times 5 \times 2 \times 5 = 2 \times 5^4$.
Now we have factorized 1250 in an easy way and factorized these factors further until we have all the factors as prime numbers.

$3388 = 847 \times 4 = 121 \times 7 \times 4 = 11 \times 11 \times 7 \times 2 \times 2 = 2^2 \times 7 \times 11^2$.

17.A Die aantal faktore van 'n positiewe heelgetal:

Dit is maklik om te sien dat 12 se faktore 1, 2, 3, 4, 6, 12 is. Dus het 12 6 faktore.

Vir groter getalle kan dit moeiliker wees en daarvoor is daar 'n baie makliker manier wat ons kan gebruik. Ons gebruik hierdie metode nou vir die getal 12:

Kry eers die priemfaktore van 12: $12 = 2 \times 2 \times 3 = 2^2 \times 3^1$.
Die aantal faktore word verkry uit $2^2 \times 3^1$. Ons gebruik 2^0, 2^1 en 2^2 en dan kan ons saam met elkeen van hulle 3^0 en 3^1 gebruik. Dit is dan $3 \times 2 = 6$ maniere en dit sê dan vir ons dat 12 ses verskillende faktore het.

1 = $2^0 \times 3^0$,
3 = $2^0 \times 3^1$,
2 = $2^1 \times 3^0$,
6 = $2^1 \times 3^1$,
4 = $2^2 \times 3^0$,
12 = $2^2 \times 3^1$

★ Kwadrate het altyd 'n onewe aantal faktore (delers):

$25 = 5^2$, en $2 + 1 = 3$; dus 3 faktore: 1, 5 en 25.

★ $96 = 48 \times 2 = 12 \times 4 \times 2 = 2^2 \times 3^1 \times 2^2 \times 2^1 = 2^5 \times 3^1$
en die aantal faktore is dan $(5+1) \times (1+1) = 6 \times 2 = 12$.

★ $960 = 96 \times 10 = 2^5 \times 3^1 \times 2^1 \times 5^1 = 2^6 \times 3^1 \times 5^1$
en die aantal faktore is dan $7 \times 2 \times 2 = 28$.

Hoeveel faktore het elk van die volgende: 128, 34, 340, 100? (Antwoorde: 8; 4; 12; 9)

17.E The number of factors of a positive integer:

It is easy to see that the factors of 12 are 1, 2, 3, 4, 6, 12.

For larger numbers it can be more difficult, but there is an easier way that we can use to find the number of factors. We will now use it for the number 12:

First, get the rpime factors for 12: $12 = 2 \times 2 \times 3 = 2^2 \times 3^1$.
The number of factors are obtained from $2^2 \times 3^1$. We use 2^0, 2^1 en 2^2 and together with each we use 3^0 and 3^1. So there are then $3 \times 2 = 6$ ways and we can say that 12 has six different factors.

1 $= 2^0 \times 3^0$,
3 $= 2^0 \times 3^1$,
2 $= 2^1 \times 3^0$,
6 $= 2^1 \times 3^1$,
4 $= 2^2 \times 3^0$,
12 $= 2^2 \times 3^1$

★ Squares always have an odd number of factors (divisors): $25 = 5^2$, and $2 + 1 = 3$; hence 3 factors: 1, 5, 25.

★ $96 = 48 \times 2 = 12 \times 4 \times 2 = 2^2 \times 3^1 \times 2^2 \times 2^1 = 2^5 \times 3^1$
and the number of factors is $(5+1) \times (1+1) = 6 \times 2 = 12$.

★ $960 = 96 \times 10 = 2^5 \times 3^1 \times 2^1 \times 5^1 = 2^6 \times 3^1 \times 5^1$
and the number of factors is $7 \times 2 \times 2 = 28$.

How many factors does each of 128, 34, 340, 100 have? (Answers: 8; 4; 12; 9)

18. 'n Drie–syfergetal (3-syfergetal) kan bv. geskryf word as $100a + 10b + c$.
A three-digit number (3-digit number) can, for example, be written as $100a + 10b + c$.

Voorbeeld/Example: $245 = 200 + 40 + 5 = 2\times100 + 4\times10 + 5 = 2\times10^2 + 4\times10^1 + 5\times10^0$

Soortgelyk vir getalle met enige aantal syfers. / Similar for numbers with any number of digits.

$24 = 2\times10^1 + 4\times10^0$,
$307 = 3\times10^2 + 0\times10^1 + 7\times10^0$.

Die getal 548 bestaan uit 3 syfers (dis 'n 3-syfergetal) en dit is 500 + 40 + 8, met ander woorde 5 honderde plus 4 tiene plus 8 ene.
The number 548 consists of 3 digits (it is a 3-digit number) and it is 500 + 40 + 8, which is 5 hundreds plus 4 tens plus 8 ones (units).

19.A 'n Getal – sy omgekeerde

Die omgekeerde van 549 is 945, en 945 – 549 = 396.
Die omgekeerde van 1258 is 8521, en 8521 – 1258 = 7263.
Meer algemeen:
Vir die getal abcd het ons 1000a+ 100b + 10c + d en sy omgekeerde is 1000d + 100c + 10b + a.
1000a – a + 100b – 10b + 10c – 100c + d – 1000d = 999a + 90b – 90c – 999a en elke term is deelbaar deur 9.

'n Getal – sy omgekeerde is deelbaar deur 9.

19.E A number – its reverse

The reverse of 549 is 945, and 945 – 549 = 396.
The reverse of 1258 is 8521, and 8521 – 1258 = 7263.

More general:
For the number abcd we have 1000a+ 100b + 10c + d and its reverse is 1000d+ 100c + 10b + a.
1000a – a + 100b – 10b + 10c – 100c + d – 1000d = 999a + 90b – 90c – 999a and each term is divisible by 9.

A number – its reverse is divisible by 9.

20.A Deelbaarheid deur 9

Om vas te stel of 'n getal deelbaar is deur 9, kan 'n mens die syfers bymekaartel en deur 9 deel en as die antwoord deelbaar is deur 9, dan is die getal deelbaar deur 9.

Daar is 'n vinniger manier:
Skrap al die 9's in die getal; skrap al die syfers wat saam 9 gee en as daar nog syfers oor is, kan jy kyk of hulle som 9 of 'n veelvoud van is.

Voorbeelde:

1. 14298945 → 142-8-45 → 142-8--- → -42----- → -42----- : 4+2 = 6 ≠ 9 sodat die getal nie deelbaar is deur 9 nie.
2. 123456789 → 12345678- → 1-3456-8- → 1-3--6-8- → --3--6--- → --------- : al die syfers is weg en die getal is dus deelbaar deur 9.
3. 142536475869709801 → 14253647586-70-801 → -42536475-6-70-801 → --2-36475-6-70-801 → --2-36-7--6-70-801 → ----36----6-70-801 → ----------6-70-801 → ----------6-70—0- → ----------6-70—0- : 6+7 = 13 en 1+3 = 4 ≠ 9 sodat die getal nie deelbaar is deur 9 nie.
4. **'n Mens hoef nie deur al die stappe te werk nie; jy kan dit sommer in een stap doen as jy wil. Ek wys dit hier in kleure, maar jy kan die syfers met potlood doodtrek soos wat jy hulle afhandel.**

   1 4 2 9 8 9 4 5 → 2 en 4 bly oor en 2 + 4 = 6 ≠ 9.

Hierdie laaste metode is lekker vinnig en baie maklik.

20.E Divisibility by 9

To determine whether a number is divisible by 9 you can add all the digits and check if that sum is divisible by 9. If the sum is divisible by 9, the number is also divisible by 9.

There is a faster way:
Delete all the 9's in the number; delete all the digits that add up to 9 and if there are any digits left you can add them and check whether their sum is 9 or a multiple of 9.

Examples:

1. 14298945 → 142-8-45 → 142-8--- → -42----- → -42----- : 4+2 = 6 ≠ 9 so that the number is not divisible by 9.
2. 123456789 → 12345678- → 1-3456-8- → 1-3--6-8- → --3--6--- → --------- : all the digits are gone and therefore the number is divisible by 9.
3. 142536475869709801 → 14253647586-70-801 → -42536475-6-70-801 → --2-36475-6-70-801 → --2-36-7--6-70-801 → ----36----6-70-801 → ----------6-70-801 → ----------6-70—0- → ----------6-70—0- : 6+7 = 13 and 1+3 = 4 ≠ 9 so that the number is not divisible by 9.
4. You do not have to work through all the steps; you can do it in one step if you want. I show it in colours below, but you can cross them out with pencil as you go along.

1 4 **2** 9 8 9 **4** 5 → 2 and 4 are left and 2 + 4 = 6 ≠ 9.

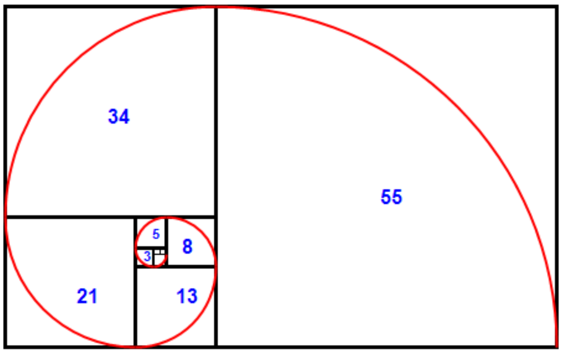
This last method is very fast and very easy.

21.A Fibonacci-getalle

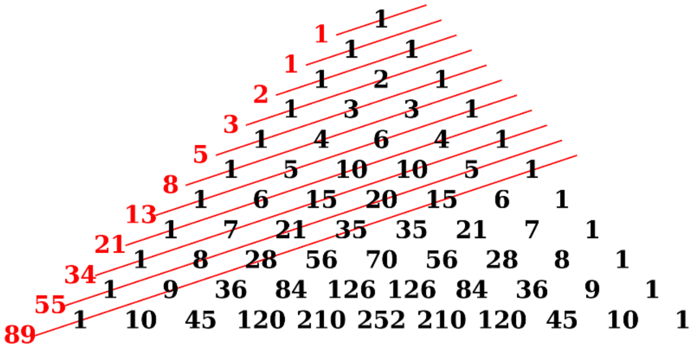
- {0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . . }
- $T_1 = 0,\ T_2 = 1,\ T_3 = 1,\ T_4 = 2,\ T_5 = 3,\ T_6 = 5,\ T_7 = 8,\ \ldots.$
- Die eerste twee terme word gegee.
- Daarvandaan is elke nuwe term die som van die vorige twee terme.
  - Bv. 21 = 13 + 8, 89 = 55 + 34, . . . , 987 = 610 + 377, . . . , 317 811 = 196 418 + 121 393, . . .
- $T_n = T_{n-1} + T_{n-2}$
  - T_{n-1} is die term net voor T_n, bv. 89
  - T_{n-2} is die term voor T_{n-1}, bv. 55
  - T_n is die term (Fibonacci-getal) wat jy wil kry: 89 + 55 = 144
- Om te toets of 'n getal, a, 'n Fibonacci-getal is, kan jy bepaal of $5a^2 + 4$ of $5a^2 - 4$ 'n kwadraat is:
  - $5 \times 89^2 + 4 = 39\,609$ wat nie 'n kwadraat is nie.
  - $5 \times 89^2 - 4 = 39\,601$ wat 'n kwadraat is en dus is 89 'n Fibonacci-getal.
  - Hierdie metode kan egter moeilik raak as jy nie 'n sakrekenaar mag gebruik nie.

21.E Fibonacci numbers

- {0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, . . . }
- $T_1 = 0,\ T_2 = 1,\ T_3 = 1,\ T_4 = 2,\ T_5 = 3,\ T_6 = 5,\ T_7 = 8,\ \ldots.$
- The first two terms are given.
- From term 3 onwards the new term is the sum of the two terms before it.
  - Bv. 21 = 13 + 8, 89 = 55 + 34, . . . , 987 = 610 + 377, . . . , 317 811 = 196 418 + 121 393, . . .
- $T_n = T_{n-1} + T_{n-2}$
  - T_{n-1} is the term just before T_n, bv. 89
  - T_{n-2} is the term before T_{n-1}, bv. 55
  - T_n is the term (Fibonacci number) that you want to find: 89 + 55 = 144
- To check if a number, a, is a Fibonacci number, you can determine if $5a^2 + 4$ or $5a^2 - 4$ is a square:
  - $5 \times 89^2 + 4 = 39\,609$ which is not a square.
  - $5 \times 89^2 - 4 = 39\,601$ which is not a square and therefore 89 is a Fibonacci number.
  - This method can get difficult if you may not use a calculator.



Fibonacci Sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...



22.1A Palindrome

'n Palindroom kan uit syfers of letters of 'n kombinasie van albei bestaan. Leestekens en spasies word geïgnoreer. Al die letters word hoofletters of kleinletters. So 'n string karakters lees dan van voor en van agter dieselfde. (Ons praat van 'n palindromiese getal/woord/string.)

Voorbeelde:

- 12321,
- ada,
- radar,
- redder,
- wow,
- No lemon, no melon.
  - Dié een sal ons skryf as "nolemonnomelon".
- Was it a cat I saw?
  - Hierdie een kan ons skryf as "wasitacatisaw".

22.2A Skryf die ry getalspalindrome as volg neer: 0,1,2,3,4,5,6,7,8,9,11,22,33, ..., 99,101,111,121, ...

Watter term?

As die palindroom 'n ewe getal syfers het, word 'n 1 vooraan die eerste helfte van die syfers gesit:

- 2332 word 12332 en 2332 is dan term nommer 123: $T_{123} = 2332$.
- 98766789 word 198766789 en 98766789 is dan term nommer 19876: Dus is $T_{19876} = 98766789$.

As die palindroom 'n onewe getal syfers het, tel jy 1 by die syfer heel links en las die res van die syfers aan tot en met die middelste syfer van die palindroom.

- 121 word 221, en 121 is dan die 22ste term in die ry: $T_{22} = 121$.
- 8206028 word 9206028 en 8206028 is dan die 9206de term in die ry:

$T_{9206} = 8206028.$

- 9230329 word 10230329 en 9230329 is dan die 1023ste term in die ry:

  $T_{1023} = 9230329.$

Aantal terme

- Daar is 9 tweesyferpalindrome: {11, 22, 33, 44, 55, 66, 77, 88, 99}.
- Daar is 90 driesyferpalindrome: {101, 111, 121, 131, 141, 151, 161, 171, 181, 191, …, 909, 919, 929, 939, 949, 959, 969, 979, 989, 999}
  - die eerste syfer kies jy uit die syfers 1 tot 9;
  - hierdie syfer is ook die derde syfer;
  - die middelste syfer kies jy uit die syfers 0 tot 9.

  Dus: 9 × 10 × 1 = 90.
- Daar is 90 viersyferpalindrome: {1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991, …, 9009, 9119, 9229, 9339, 9449, 9559, 9669, 9779, 9889, 9999}
  - die eerste syfer kies jy uit die syfers 1 tot 9;
  - hierdie syfer is ook die vierde syfer;
  - die tweede syfer kies jy uit die syfers 0 tot 9.
  - hierdie syfer is ook die derde syfer.

  Dus: 9 × 10 × 1 × 1 = 90.

Op hierdie manier is dit eintlik taamlik maklik om die aantal palindrome vir 'n spesifieke omvang te bereken.

22.3A Nog oor palindrome

Palindromiese getalle wat ook priemgetalle is: {2, 3, 5, 7, 11, 101, 131, 151, . . .}

Palindromiese getalle wat ook kwadrate is: {0, 1, 4, 9, 121, 484, 676, 10201, 12321, . . .}

Palindromiese getalle wat ook derdemagte is: {0, 1, 14641, . . .}

Die volgende 9 terme is ook palindrome:

$1^2 = 1$
$11^2 = 121$
$111^2 = 12\ 321$
$1\ 111^2 = 1\ 234\ 321$
. . .
$1\ 111\ 111^2 = 1\ 234\ 567\ 654\ 321$
. . .
$111\ 111\ 111^2 = 12\ 345\ 678\ 987\ 654\ 321$

Die produk van twee palindrome is nie altyd 'n palindroom nie.
77 × 88 = 6776, maar 11 × 55 = 605.

Daar is 'n paar palindrome wat die produk van opeenvolgende priemgetalle is:
7 × 11 × 13 = 1001,
5 × 7 × 11 × 13 = 5005 = 5 × 1001,
7 × 11 × 13 × 17 × 19 = 323 323,
17 ×19 = 323

Herhaaldelike som van getalle

Wanneer jy 'n getal en sy omgekeerde bymekaartel, en jy herhaal hierdie proses met die antwoord, kry jy uiteindelik 'n palindromiese getal.

1. 578 + 875 = 1453
   1453 + 3541 = 4994   578 het net twee stappe nodig gehad.
2. 89 + 98 = 187
   187 + 781 = 968
   968 + 869 = 1837
   1837 + 7381 = 9218
   . . .
   . . . + . . . = 8 813 200 023 188   89 het 24 stappe nodig.

- - - - - - - - - - - - - -

22.1E Palindromes

A palindrome can consist of digits or letters or a combination of both. Punctuation marks and spaces are ignored. All the letters become capital letters or small letters. Such a string of characters then reads the same forwards and backwards.
(We talk about a palindromic number/word/string.)

Examples:

- 12321,
- ada,
- radar,
- redder,
- wow,
- No lemon, no melon.
  - This one will be written as "nolemonnomelon".
- Was it a cat I saw?
  - This one will be written as "wasitacatisaw".

22.2E Write down the sequence of number palindromes: 0,1,2,3,4,5,6,7,8,9,11,22,33, ..., 99,101,111,121, ...

Which term?

If a palindrome has an even number of digits, add a 1 at the front of the first half of the digits:

- 2332 becomes 12332 and 2332 is then term number 123: $T_{123} = 2332$.
- 98766789 becomes 198766789 and 98766789 is then term number 19876: Therefore,
  $T_{19876} = 98766789$.

If a palindrome has an odd number of digits, add 1 to the digit on the left and add it to the rest of the digits up to and indlucing the digit in the middle of the palindrome.

- 121 becomes 221, and 121 is then the 22nd term in the sequence: $T_{22} = 121$.
- 8206028 becomes 9206028 and 8206028 is then the 9206th term in the sequence:
  $T_{9206} = 8206028$.
- 9230329 becomes 10230329 and 9230329 is then the 1023rd term in the sequence:
  $T_{1023} = 9230329$.

Number of terms (How many terms?)

- There are 9 two-digit palindromes: {11, 22, 33, 44, 55, 66, 77, 88, 99}.
- There are 90 three-digit palindromes: {101, 111, 121, 131, 141, 151, 161, 171, 181, 191, …, 909, 919, 929, 939, 949, 959, 969, 979, 989, 999}

- the first digit is selected from the digits 1 to 9;
- this digit is also the third digit;
- the middle digit is selected from the digits 0 to 9.

Therefore: $9 \times 10 \times 1 = 90$.

- There are 90 four-digit palindromes: {1001, 1111, 1221, 1331, 1441, 1551, 1661, 1771, 1881, 1991, …, 9009, 9119, 9229, 9339, 9449, 9559, 9669, 9779, 9889, 9999}
  - the first digit is selected from the digits 1 to 9;
  - this digit is also the fourth digit;
  - the second digit is selected from the digits 0 to 9.
  - this digit is also the third digit.

  Therefore: $9 \times 10 \times 1 \times 1 = 90$.

In this way it is actually quite easy to determine the number of palindromes in a specified range.

22.3E More about palindromes

Palindromic numbers that are also prime numbers: {2, 3, 5, 7, 11, 101, 131, 151, . . .}

Palindromic numbers that are also squares: {0, 1, 4, 9, 121, 484, 676, 10201, 12321, . . .}

Palindromic numbers that are also cubes: {0, 1, 14641, . . .}

The following 9 terms are also palindromes:

$1^2 = 1$
$11^2 = 121$
$111^2 = 12\,321$
$1\,111^2 = 1\,234\,321$
. . .
$1\,111\,111^2 = 1\,234\,567\,654\,321$
. . .
$111\,111\,111^2 = 12\,345\,678\,987\,654\,321$

The product of two palindromes is not always a palindrome.
$77 \times 88 = 6776$, maar $11 \times 55 = 605$.

There are some palindromes that are the product of consecutive prime numbers:
$7 \times 11 \times 13 = 1001$,
$5 \times 7 \times 11 \times 13 = 5005 = 5 \times 1001$,
$7 \times 11 \times 13 \times 17 \times 19 = 323\,323$,
$17 \times 19 = 323$

Repeated sum of numbers

If you add a number and its reverse and repeat this process, it will eventually end in a palindromic number.

1. 578 + 875 = 1453
   1453 + 3541 = 4994 578 only needed two steps.
2. 89 + 98 = 187
   187 + 781 = 968
   968 + 869 = 1837
   1837 + 7381 = 9218
   . . .

. . . + . . . = 8 813 200 023 188 89 needed 24 steps.

23. Persentasievermeerdering of –vermindering / Percentage increase or decrease

- Vermeerder 20 met 10%: / Increase 20 with 10%:

$$20 \times 10\% + 20 = 20(10\% + 1) = 20(0,1 + 1) = 20(1,1) = 22$$

wat dieselfde is as / which is the same as

$$20 \times (100 + 10)\% = 20 \times 110\% = 20 \times 1,1 = 22$$

- Vermeerder 15 met 15%: $15 \times 1.15 = 17.25$ / Increase 15 with 15%: $15 \times 1.15 = 17.25$
- Verminder 55 met 15%: / Decrease 55 with 15%:

$$55 - 55 \times 15\% = 55(1 - 15\%) = 55(1 - 0,15) = 55(0,85) = 46,75$$

wat dieselfde is as / which is the same as

$$55 \times (100 - 15)\% = 55 \times 85\% = 55 \times 0,85$$

- Vermminder 15 met 15%: $15 \times 0.85 = 12.75$ / Decrease 15 with 15%: $15 \times 0.85 = 12.75$
- Vermeerder R200 met 10% en verlaag (verminder) die nuwe prys dan met 10%:
  Die nuwe verhoogde prys is dan $200 \times 110\% =$ R220.
  Die uiteindelike prys na 10% verlaging is dan $220 \times 90\% =$ R198.
- Increase R200 with 10% and then decrease this new price by 10%:
  The new increased price is then $200 \times 110\% =$ R220.
  And the final price after a decrease of 10% on the higher price is $220 \times 90\% =$ R198.

24. Eksponente / Exponents

★ $2^3 \times 2^5 = 2^{3+5} = 2^8$

★ $2^{3/4} \times 2^{5/4} = 2^{3/4+5/4} = 2^{8/4} = 2^2$

★ $(2^3)^5 = 2^{3\times5} = 2^{15}$

★ $2^5 \div 2^3 = 2^{5-3} = 2^2$

★ $2^3 \div 2^5 = 2^{3-5} = 2^{-2}$

★ $2^{5/4} \div 2^{3/4} = 2^{5/4-3/4} = 2^{2/4} = 2^{1/2}$

★ $2^{1/2} = \sqrt{2} = \sqrt[2]{2}$

★ $16^{1/2} = \sqrt{16} = 4$

★ $27^{1/3} = \sqrt[3]{27} = 3$

☺ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

☺ $\frac{1}{3^{-4}} = 3^4 = 81$

25.A Fakulteite

$n! = n(n-1)(n-2)\ldots1$

bv. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

25.1A Aantal nulle

- $5! = \underline{5} \times 4 \times 3 \times \underline{2} \times 1 = 120$ eindig in een nul; daar is altyd meer 2's as 5'e; ons tel dus hoeveel 5'e daar is
  * $5 \times 2 = 10$
  * $1! = 1$ en $0! = 1$

- $10! = \underline{10} \times 9 \times 8 \times 7 \times 6 \times \underline{5} \times 4 \times 3 \times \underline{2} \times 1$. . . . . eindig in twee nulle

Hierdie metode is egter omslagtig wanneer die getalle groot is. Dan is die volgende metode baie meer doeltreffend:
Ons gebruik "heelgetaldeling" (ons gooi die breukgedeeltes weg):

- In hoeveel nulle eindig 100! ?

100 / 5 = 20	100 / 5 = 20
20 / 5 = 4	100 / 25 = 4
∴ 20 + 4 = 24 nulle	∴ 20 + 4 = 24 nulle

- In hoeveel nulle eindig 11! ?
  11 / 5 = 2 nulle

- In hoeveel nulle eindig 24! ?
  24 / 5 = 4 nulle

- In hoeveel nulle eindig 240! ?

240 / 5 = 48	of	240 / 5 = 48
48 / 5 = 9		240 / 25 = 9
9 / 5 = 1		240 / 125 = 1

∴ 48 + 9 + 1 = 58 nulle

25.E Factorials

$n! = n(n-1)(n-2)\ldots1$

e.g. $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

25.1E Number of zeros

- $5! = \underline{5} \times 4 \times 3 \times \underline{2} \times 1 = 120$ ends in one zero; there are always more 2's than 5's; hence we count the number of 5's.
  * $5 \times 2 = 10$
  * $1! = 1$ and $0! = 1$

- $10! = \underline{10} \times 9 \times 8 \times 7 \times 6 \times \underline{5} \times 4 \times 3 \times \underline{2} \times 1$. . . . . ends in two zeros

For the following we use "integer division" (we throw the fraction parts away):

- In how many zeros does 100! end?

100 / 5 = 20	100 / 5 = 20
20 / 5 = 4	100 / 25 = 4
∴ 20 + 4 = 24 zeros	∴ 20 + 4 = 24 zeros

- In how many zeros does 11! end?
  11 / 5 = 2 zeros
- In how many zeros does 24! end?
  24 / 5 = 4 zeros
- In how many zeros does 240! end?

240 / 5 = 48	of / or	240 / 5 = 48
48 / 5 = 9		240 / 25 = 9
9 / 5 = 1		240 / 125 = 1

$\therefore 48 + 9 + 1 = 58$ zeros

26.A Aantal kwadrate en derdemagte

'n Volkome vierkant is 'n getal waarvan die vierkantswortel 'n heelgetal is, bv. 1, 4, 9, ens. Netso vir volkome derdemagte en ander magte.

1. Hoeveel kwadrate (volkome vierkante) is daar in 1!·2!·3!·...·9!

Oplossing:
$2! = 2$
$3! = 2 \times 3$
$4! = 4 \times 3 \times 2$
$5! = 5 \times 4 \times 3 \times 2$
$6! = 6 \times 5 \times 4 \times 3 \times 2$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

Die produk is dan $2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9$
$= 2^8 \times 3^7 \times (2^2)^6 \times 5^5 \times (2 \times 3)^4 \times 7^3 \times (2^3)^2 \times 3^2$
$= 2^8 \times 3^7 \times 2^{12} \times 5^5 \times 2^4 \times 3^4 \times 7^3 \times 2^6 \times 3^2$
$= 2^{30} \times 3^{13} \times 5^5 \times 7^3$
$= (2^2)^{15} \times (3^2)^6 \times 3 \times (5^2)^2 \times 5 \times (7^2)^1 \times 7$

Daar is dus $(15 + 1)(6 + 1)(2 + 1)(1 + 1) = 16 \times 7 \times 3 \times 2 = 672$ kwadrate in 1!·2!·3!·...·9!

2. Hoeveel derdemagte is daar in 1!·2!·3!·...·9!

Oplossing:
$2! = 2$
$3! = 2 \times 3$
$4! = 4 \times 3 \times 2$
$5! = 5 \times 4 \times 3 \times 2$
$6! = 6 \times 5 \times 4 \times 3 \times 2$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

Die produk is dan $2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9$
$= 2^8 \times 3^7 \times (2^2)^6 \times 5^5 \times (2 \times 3)^4 \times 7^3 \times (2^3)^2 \times 3^2$
$= 2^8 \times 3^7 \times 2^{12} \times 5^5 \times 2^4 \times 3^4 \times 7^3 \times 2^6 \times 3^2$
$= 2^{30} \times 3^{13} \times 5^5 \times 7^3$
$= (2^3)^{10} \times (3^3)^4 \times 3 \times (5^3)^1 \times 5^2 \times (7^3)^1$

Daar is dus $(\mathbf{10} + 1)(\mathbf{4} + 1)(\mathbf{1} + 1)(\mathbf{1} + 1) = 11 \times 5 \times 2 \times 2 = 220$ derdemagte in $1!\cdot 2!\cdot 3!\cdot ...\cdot 9!$

Die metode is dus dieselfde vir die aantal faktore, die aantal kwadrate, die aantal derdemagte en vir enige mag verder aan.

3. Hoeveel kwadrate (volkome vierkante) is daar in 960?
Skryf 960 as die produk van sy priemfaktore.

$960 = 2^6 \times 3^1 \times 5^1$
$= (2^2)^3 \times 3^1 \times 5^1$

Daar is dus $(3 + 1) = 4$ volkome vierkante (kwadrate) in 960.
Jy kan hulle neerskryf: $(2^2)^0 = 1$, $(2^2)^1 = 4$, $(2^2)^2 = 16$, $(2^2)^3 = 32$.

26.E Number of squares and cubes

A perfect square is a number of which the square root is an integer, e.g. 1, 4, 9, etc. Similar for perfect cubes and other powers.

1. How many perfect squares are there in $1!\cdot 2!\cdot 3!\cdot ...\cdot 9!$?

Solution:
$2! = 2$
$3! = 2 \times 3$
$4! = 4 \times 3 \times 2$
$5! = 5 \times 4 \times 3 \times 2$
$6! = 6 \times 5 \times 4 \times 3 \times 2$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

The product is then $2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9$
$= 2^8 \times 3^7 \times (2^2)^6 \times 5^5 \times (2 \times 3)^4 \times 7^3 \times (2^3)^2 \times 3^2$
$= 2^8 \times 3^7 \times 2^{12} \times 5^5 \times 2^4 \times 3^4 \times 7^3 \times 2^6 \times 3^2$
$= 2^{30} \times 3^{13} \times 5^5 \times 7^3$
$= (2^2)^{\mathbf{15}} \times (3^2)^{\mathbf{6}} \times 3 \times (5^2)^{\mathbf{2}} \times 5 \times (7^2)^{\mathbf{1}} \times 7$

Therefore there are $(\mathbf{15} + 1)(\mathbf{6} + 1)(\mathbf{2} + 1)(\mathbf{1} + 1) = 16 \times 7 \times 3 \times 2 = 672$ squares in $1!\cdot 2!\cdot 3!\cdot ...\cdot 9!$

2. How many cubes are there in $1!\cdot 2!\cdot 3!\cdot ...\cdot 9!$?

Solution:
$2! = 2$
$3! = 2 \times 3$
$4! = 4 \times 3 \times 2$
$5! = 5 \times 4 \times 3 \times 2$
$6! = 6 \times 5 \times 4 \times 3 \times 2$
$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$8! = 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$
$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2$

The product is then $2^8 \times 3^7 \times 4^6 \times 5^5 \times 6^4 \times 7^3 \times 8^2 \times 9$
$= 2^8 \times 3^7 \times (2^2)^6 \times 5^5 \times (2 \times 3)^4 \times 7^3 \times (2^3)^2 \times 3^2$
$= 2^8 \times 3^7 \times 2^{12} \times 5^5 \times 2^4 \times 3^4 \times 7^3 \times 2^6 \times 3^2$
$= 2^{30} \times 3^{13} \times 5^5 \times 7^3$
$= (2^3)^{\mathbf{10}} \times (3^3)^{\mathbf{4}} \times 3 \times (5^3)^{\mathbf{1}} \times 5^2 \times (7^3)^{\mathbf{1}}$

Therefore there are (**10** + 1)(**4** + 1)(**1** + 1)(**1** + 1) = 11 × 5 × 2 × 2 = 220 cubes in 1!·2!·3!·...·9!

The method are the same for the number of factors, the number of squares, the number of cubes and for any power further on.

3. How many perfect squares are there in 960?
Write 960 as the product of its prime factors.

$960 = 2^6 \times 3^1 \times 5^1$
$= (2^2)^3 \times 3^1 \times 5^1$

Therefore there are (3 + 1) = 4 perfect squares in 960.
You can write them down: $(2^2)^0 = 1$, $(2^2)^1 = 4$, $(2^2)^2 = 16$, $(2^2)^3 = 32$.

27. Bereken die wortels met behulp van priemfaktore / Calculate the roots using prime factors

$\sqrt{144} = \sqrt{2^4 \times 3^2} = (2^4 \times 3^2)^{1/2} = 2^2 \times 3 = 12$ vierkantswortel / square root

$\sqrt{324} = \sqrt{2^2 \times 3^4} = (2^2 \times 3^4)^{1/2} = 2 \times 3^2 = 2 \times 9 = 18$ vierkantswortel / square root

$\sqrt[3]{1728} = \sqrt[3]{2^6 \times 3^3} = (2^6 \times 3^3)^{1/3} = 2^2 \times 3 = 12$ derdemagswortel / cube root

$\sqrt[6]{15625} = \sqrt[6]{5^6} = (5^6)^{1/6} = 5$ sesdemagswortel / sixth root

28. Skryf al die getalle met dieselfde grondtal: / Write all numbers with the same base:

$\frac{1}{2}$ van/of $4^{15} = \frac{1}{2} \times (2^2)^{15} = \frac{1}{2} \times 2^{30} = 2^{-1} \times 2^{30} = 2^{30-1} = 2^{29}$. grondtal = 2/base = 2

$2^{-3} \div \frac{1}{2} = 2^{-3} \div 2^{-1} = 2^{-3} \times 2^1 = 2^{-2} = \frac{1}{2^2}$ grondtal = 2/base = 2

16 ÷ ½ = 32 en 'n mens kan sê dit beteken dat daar 32 halwes in 16 is.

16 ÷ ½ = 32 and one can say that it means that there are 32 halves in 16.

29. Eenvoudigste wortelvorm / Simplest surd form

Probeer die getal skryf as 'n (kwadraat × iets).

Byvoorbeeld, 8 = 4 × 2, 112 = 16 × 7, 25 = 25 × 1.

Try to write the number as a (square × something).

For example, 8 = 4 × 2, 112 = 16 × 7, 25 = 25 × 1.

★ $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \times \sqrt{2} = 2\sqrt{2}$

★ $\sqrt{128} = \sqrt{64 \times 2} = \sqrt{64} \times \sqrt{2} = 8\sqrt{2}$

★ $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

30. Rasionaliseer die noemer of die teller / Rationalize the denominator or the numerator

Onthou dat / Remember that:

★ $\sqrt{2} \times \sqrt{2} = 2$

★ $\sqrt{4} \times \sqrt{4} = 4$

★ $\sqrt{3} \times \sqrt{3} = 3$

♦ $(-1)^2 = (-1) \times (-1) = 1$

♦ $-(1)^2 = -(1) \times (1) = -1$

☺ $(12 - 11)(12 + 11) = 1 \times 23 = 23$

☺ $(\sqrt{4} - \sqrt{3})(\sqrt{4} + \sqrt{3})$
$= \sqrt{4} \times \sqrt{4} + \sqrt{4} \times \sqrt{3} - \sqrt{3} \times \sqrt{4} - \sqrt{3} \times \sqrt{3}$
$= 4 + \sqrt{12} - \sqrt{12} - 3$
$= 1$

☺ $(\sqrt{16} - \sqrt{3})(\sqrt{16} + \sqrt{3})$
$= \sqrt{16} \times \sqrt{16} + \sqrt{16} \times \sqrt{3} - \sqrt{3} \times \sqrt{16} - \sqrt{3} \times \sqrt{3}$
$= 16 + \sqrt{48} - \sqrt{48} - 3$
$= 13$

♦ Vereenvoudig / Simplify:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}}$$

$$= \frac{1}{\sqrt{1}+\sqrt{2}} \times \frac{\sqrt{1}-\sqrt{2}}{\sqrt{1}-\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} \times \frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$

$$= \frac{\sqrt{1}-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3}$$

$$= -\sqrt{1} + \sqrt{2} - \sqrt{2} + \sqrt{3}$$

$$= -1 + \sqrt{3}$$

♦ Watter getal is die grootste? / Which number is the larger?

$\sqrt{21} - \sqrt{20}$ of/or $\sqrt{20} - \sqrt{19}$:

Gebruik $(\sqrt{21} - \sqrt{20})$ en vermenigvuldig dit met $\frac{(\sqrt{21}+\sqrt{20})}{\sqrt{21}+\sqrt{20}}$.
Use $(\sqrt{21} - \sqrt{20})$ and multiply it by $\frac{(\sqrt{21}+\sqrt{20})}{\sqrt{21}+\sqrt{20}}$.

$$\frac{(\sqrt{21}-\sqrt{20})(\sqrt{21}+\sqrt{20})}{\sqrt{21}+\sqrt{20}} = \frac{1}{\sqrt{21}+\sqrt{20}}:$$

Gebruik $(\sqrt{20} - \sqrt{19})$ en vermenigvuldig dit met $\frac{(\sqrt{20}+\sqrt{19})}{\sqrt{20}+\sqrt{19}}$.
Use $(\sqrt{20} - \sqrt{19})$ and multiply it by $\frac{(\sqrt{20}+\sqrt{19})}{\sqrt{20}+\sqrt{19}}$.

$$\frac{(\sqrt{20}-\sqrt{19})(\sqrt{20}+\sqrt{19})}{\sqrt{20}+\sqrt{19}} = \frac{1}{\sqrt{20}+\sqrt{19}}:$$

Die tellers is dieselfde, en dus sal die breuk met die grootste noemer die kleinste wees.

[$\frac{3}{4} > \frac{3}{8}$; $\frac{1}{72} < \frac{1}{60}$]

The numerators are the same, and therefore the fraction with the largest denominator will be the smallest. [$\frac{3}{4} > \frac{3}{8}$; $\frac{1}{72} < \frac{1}{60}$]

$\sqrt{20} - \sqrt{19} > \sqrt{21} - \sqrt{20}$
[0.11323701145890584058136535360294 > 0.11043973995626061376969985626546]

Let op: / Note that: $\sqrt{2} - \sqrt{1} > \sqrt{3} - \sqrt{2} > \sqrt{4} - \sqrt{3} > \sqrt{5} - \sqrt{4} > \ldots$

Einde van Deel 1 / End of Part 1