ALGEMEEN (2) Ken so goed as moontlik vir deelname aan kompetisies.
GENERAL (2) Know as well as possible for participation in competitions.

ALGEMEEN 2 / GENERAL 2 Gr 7 - 9

GETALLE / NUMBERS

1.A Leer jou tafels: 1×1 tot by 12×12

Asook ander wat gereeld voorkom, bv. 13×13, 14×14, 15×15, 16×16, 17×17, 18×18, 19×19, 20×20, 21×21, 30×30, 31×31, . . .

Veelvoude van 11: $11 \times 12 = 132$, $11 \times 13 = 143$, $11 \times 14 = 154$, . . .

Ook handig is $42^2 = 1764$, $43^2 = 1849$, $44^2 = 1936$, $45^2 = 2015$, $46^2 = 2116$ (die kwadrate rondom die huidige jaartal)

Derdemagte: $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 2^6 = 64$, $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, . . .

$13^2 = 169$, $31^2 = 961$

1.E Study your times tables: 1×1 to 12×12

As well as others that occur often, e.g. 13×13, 14×14, 15×15, 16×16, 17×17, 18×18, 19×19, 20×20, 21×21, 30×30, 31×31, . . .

Multiples of 11: $11 \times 12 = 132$, $11 \times 13 = 143$, $11 \times 14 = 154$, . . .

Also useful are $42^2 = 1764$, $43^2 = 1849$, $44^2 = 1936$, $45^2 = 2015$, $46^2 = 2116$ (the squares around the current year)

Cubes: $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 2^6 = 64$, $5^3 = 125$, $6^3 = 216$, $7^3 = 343$, . . .

$13^2 = 169$, $31^2 = 961$

2. Leer / Study:

$\frac{1}{5} = 0,2$ $\frac{2}{5} = 0,4$ $\frac{3}{5} = 0,6$ $\frac{4}{5} = 0,8$

$\frac{1}{2} = 0,5$

$\frac{1}{4} = 0,25$ $\frac{3}{4} = 0,75$

$\frac{1}{8} = 0,125$ $\frac{3}{8} = 0,375$ $\frac{5}{8} = 0,625$ $\frac{7}{8} = 0,875$

$\frac{1}{9} = 0.\dot{1}$ $\frac{2}{9} = 0.\dot{2}$ $\frac{3}{9} = \frac{1}{3} = 0.\dot{3}$ $\frac{4}{9} = 0.\dot{4}$ $\frac{5}{9} = 0.\dot{5}$ $\frac{6}{9} = \frac{2}{3} = 0.\dot{6}$

$\frac{7}{9} = 0.\dot{7}$ $\frac{8}{9} = 0.\dot{8}$ $\frac{9}{9} = 0.\dot{9} = 1$

3.A Intervalle

Intervalle word saam met die reële getalle gebruik.

Let op hoe die ronde hakies ( ) en die vierkantige hakies [ ] gebruik word:

As die getal ingesluit is, word vierkantige hakies gebruik; as die getal nie ingesluit is nie word ronde hakies gebruik.

- die reële getalle van 2 tot 10 wat geskryf kan word as
  - $x \in [2; 10]$ of
  - $2 \leq x \leq 10$

  * 2 en 10 is albei ingesluit

  * geslote interval

- die reële getalle tussen 2 en 10 wat geskryf kan word as
  - $x \in (2; 10)$ of
  - $2 < x < 10$

  * 2 en 10 is albei uitgesluit

  * oop interval

- die reële getalle van 2 tot voor 10 wat geskryf kan word as
  - $x \in [2; 10)$ of
  - $2 \leq x < 10$

  * 2 is ingesluit, maar 10 is uitgesluit

  * half-oop interval

- die reële getalle van net na 2 tot 10 wat geskryf kan word as
  - $x \in (2; 10]$ of
  - $2 < x \leq 10$

  * 2 self is uitgesluit, maar 10 is ingesluit

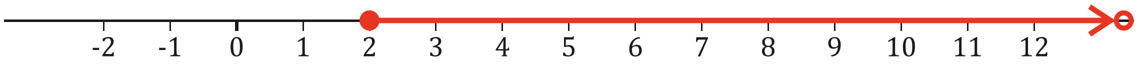
  * half-oop interval

- die reële getalle van 2 tot oneindig wat geskryf kan word as
  - $x \in [2; \infty)$ of

  - $2 \leq x < \infty$

  * 2 is <u>ingesluit</u>, oneindig is altyd uitgesluit omdat dit nie 'n vaste posisie op die getallelyn het nie.

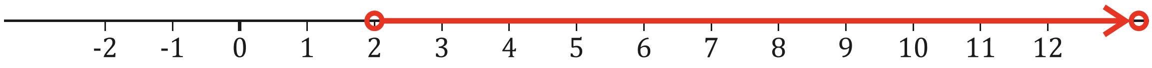
  * geslote straal



- die reële getalle van net na 2 tot oneindig wat geskryf kan word as
  - $x \in (2; \infty)$ of
  - $2 < x < \infty$

  * 2 is <u>uitgesluit</u>, oneindig is altyd uitgesluit.

  * oop straal



- die reële getalle tussen $-\infty$ en 10 wat geskryf kan word as
  - $x \in (-\infty; 10)$ of
  - $-\infty < x < 10$

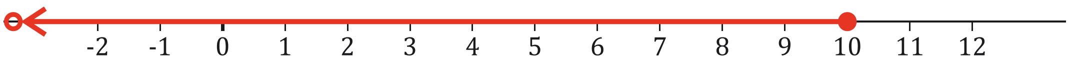
  * $-\infty$ en 10 is albei uitgesluit

  * oop straal

- die reële getalle van $-\infty$ tot 10 wat geskryf kan word as
  - $x \in (-\infty; 10]$ of
  - $-\infty < x \leq 10$

  * $-\infty$ is uitgesluit, maar 10 is ingesluit

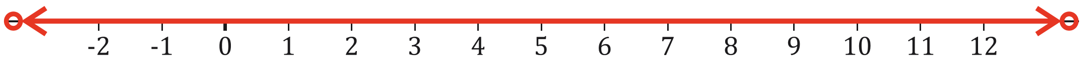
  * geslote straal



- die reële getalle tussen $-\infty$ en ∞ wat geskryf kan word as
  - $x \in (\infty; 10)$ of
  - $-\infty < x < \infty$

  * $-\infty$ en ∞ is albei uitgesluit

  * oop straal



3.E Intervals

Intervals are used with real numbers.

Note how the square [ ] and round ( ) brackets are used.:

If the number is included, we use square brackets; if the number is not included we use round brackets.

- the real numbers from 2 to 10 which can be written as
  - $x \in [2; 10]$ or
  - $2 \leq x \leq 10$

  * 2 and 10 are both included

  * closed interval

- the real numbers between 2 and 10 which can be written as
  - $x \in (2; 10)$ or
  - $2 < x < 10$

  * 2 and 10 are both excluded

  * open interval

- the real numbers from 2 to just before 10 which can be written as
  - $x \in [2; 10)$ or
  - $2 \leq x < 10$

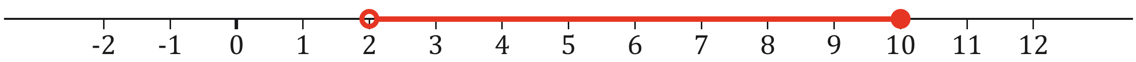
  * 2 is included, 10 is excluded

  * half-open interval

- the real numbers from just after 2 to 10 which can be written as
  - $x \in (2; 10]$ or
  - $2 < x \leq 10$

  * 2 is excluded, 10 is included

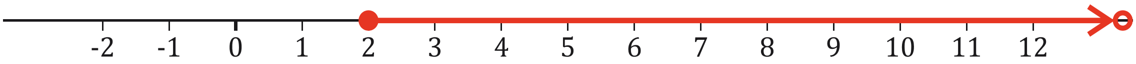
  * half-open interval



- the real numbers from 2 to infinity which can be written as
  - $x \in [2; \infty)$ or
  - $2 \leq x < \infty$

  * 2 is <u>included</u>, infinity is always excluded since it has no fixed point on the number line.

  * closed ray



- the real numbers from just after 2 to infinity which can be written as
  - $x \in (2; \infty)$ or
  - $2 < x < \infty$

  * 2 is <u>excluded</u>, infinity is always excluded.

  * open ray

- the real numbers between $-\infty$ and 10: $x \in (-\infty; 10)$ of $-\infty < x < 10$

  * $-\infty$ and 10 are both excluded

  * open ray

- the real numbers from $-\infty$ to 10 which can be written as
  - $x \in (-\infty; 10]$ or
  - $-\infty < x \leq 10$

  * $-\infty$ is excluded, 10 is included

  * closed ray

- the real numbers between $-\infty$ and ∞ which can be written as
  - $x \in (\infty; 10)$ or
  - $-\infty < x < \infty$

  * $-\infty$ and ∞ are both excluded

  * open ray

4.A Absolute waarde

Dit maak nie saak of 'n getal positief, nul of negatief is nie: sy absolute waarde is altyd positief.

Ons skryf dan die getal waarvan die absolute waarde gekry moet word tussen regop strepies.

Ons werk hier ook met reële getalle.

$|-3| = 3$

$\left|-\frac{22}{35}\right| = \frac{22}{35}$

$|-0,\dot{7}\dot{2}| = 0,\dot{7}\dot{2}$

$|6| = 6$

$\lvert x\rvert = 0$ as $x = 0$	
$\lvert x\rvert = x$ as $x > 0$	$\lvert 35\rvert = 35$
$\lvert x\rvert = -x$ as $x < 0$	$\lvert -35\rvert = -(-35) = 35$

$|x| < 0$ is nie moontlik vir enige waarde van *x* nie.

♦♦ Los op vir *x* in elk van die volgende gevalle:

i $|x - 2| = 5$

Dus: *x* = 7 of *x* = –3.

$x \in \{-3;\ 7\}$

ii $|x - 2| < 5$

$-5 < x - 2 < 5$

x is dus op die getallelyn tussen –3 en 7. Die getalle –3 en 7 is nie ingesluit nie. Dit kan geskryf word as

- $-3 < x < 7$ of
- $x \in (-3; 7)$

iii $|x - 2| \leq 5$

$-5 \leq x - 2 \leq 5$

x is dus op die getallelyn tussen –3 en 7. Die getalle –3 en 7 is ingesluit. Dit kan geskryf word as

- $-3 \leq x \leq 7$ of
- $x \in [-3; 7]$

Toets verskillende waardes van *x*:
As $x = -4$, dan is $|-4 - 2| = |-6| = 6$ en dit is groter as 5. ✗
As $x = -1$, dan is $|-1 - 2| = |-3| = 3 \leq 5$. ✓
As $x = 9$, dan is $|9 - 2| = |7| = 7$ en dit is groter as 5. ✗
Slegs $x \in [-3; 7]$ is geldig. As $x < -3$ of $x > 7$ dan is die oplossings ongeldig.

iv $|x-2|>5$

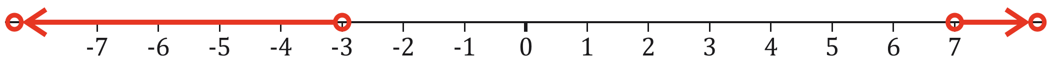
$x-2>5$ of $x-2<-5$

$x>7$ of $x<-3$

Toets verskillende waardes van *x*:
As $x=-4$, dan is $|-4-2|=|-6|=6>5$. ✓
As $x=-1$, dan is $|-1-2|=|-3|=3\leq 5$. ✗
As $x=9$, dan is $|9-2|=|7|=7>5$. ✓
$x\in[-3;7]$ is ongeldig. As $x<-3$ of $x>7$ dan is die oplossings geldig.

x is dus op die getallelyn buite die getalle –3 en 7. Die getalle –3 en 7 is nie ingesluit nie. Dit kan geskryf word as

- $x<-3$ of $x>7$ of
- $x\in(-\infty;-3)\cup(7;\infty)$

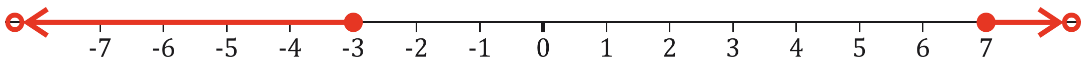


v $|x-2|\geq 5$

$x-2\geq 5$ of $x-2\leq -5$

x is dus op die getallelyn buite die getalle –3 en 7. Die getalle –3 en 7 is ingesluit. Dit kan geskryf word as

- $x\leq -3$ of $x\geq 7$ of
- $x\in(-\infty;-3]\cup[7;\infty)$



4.E Absolute value

It does not matter whether a number is positive, zero or negative: its absolute value is always positive.

We write the number of which we want to find the absolute value, between two vertical lines.

Here we also work with real numbers.

$|-3|=3$

$\left|-\frac{22}{35}\right|=\frac{22}{35}$

$|-0,\dot{7}\dot{2}|=0,\dot{7}\dot{2}$

$|6|=6$

$\lvert x\rvert = 0$ as $x = 0$	
$\lvert x\rvert = x$ as $x > 0$	$\lvert 35\rvert = 35$
$\lvert x\rvert = -x$ as $x < 0$	$\lvert -35\rvert = -(-35) = 35$

$|x| < 0$ is not possible for any value of x.

♦♦ Solve for x in each of the following cases:

i $|x-2| = 5$

Therefore $x = 7$ or $x = -3$.

$x \in \{-3;\ 7\}$

ii $|x-2| < 5$

$-5 < x-2 < 5$

x is on the number line between –3 and 7. The numbers–3 and 7 are not included. It can be written as

- $-3 < x < 7$ or
- $x \in (-3; 7)$

iii $|x-2| \leq 5$

$-5 \leq x-2 \leq 5$

x is on the number line between –3 and 7. The numbers –3 and 7 are included. It can be written as

- $-3 \leq x \leq 7$ of
- $x \in [-3; 7]$

Test for different values of x:
If $x = -4$, then $|-4-2| = |-6| = 6$ but it is greater than 5. ✗
If $x = -1$, then $|-1-2| = |-3| = 3 \leq 5$. ✓
If $x = 9$, then $|9-2| = |7| = 7$ but it is greater than 5. ✗

Only $x \in [-3; 7]$ are valid. If $x < -3$ or $x > 7$ the solutions are invalid.

iv $|x-2| > 5$

$x-2 > 5$ or $x-2 < -5$

Toets verskillende waardes van x:

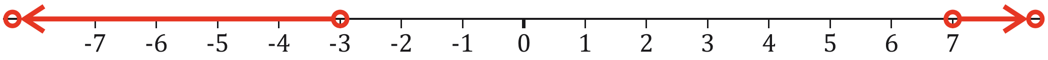
If $x = -4$, then $|-4-2| = |-6| = 6 > 5$. ✓
If $x = -1$, then $|-1-2| = |-3| = 3 \leq 5$. ✗
If $x = 9$, then $|9-2| = |7| = 7 > 5$. ✓

$x \in [-3; 7]$ are not valid (these values are invalid). If $x < -3$ or $x > 7$ the solutions are valid.

x is on the number line outside the numbers –3 and 7. The numbers –3 and 7 are not indcluded.

It can be written as

- $x < -3$ of $x > 7$ of
- $x \in (-\infty; -3) \cup (7; \infty)$

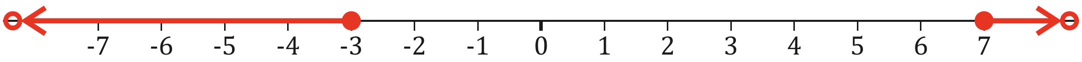


v $|x - 2| \geq 5$

$x - 2 \geq 5$ of $x - 2 \leq -5$

x is on the number line outside the numbers –3 en 7. The numbers –3 and 7 are included. It can be written as

- $x \leq -3$ of $x \geq 7$ of
- $x \in (-\infty; -3] \cup [7; \infty)$



5.A Plafon en Vloer

Die plafonfunksie $\lceil\ \rceil$ neem 'n reële getal as invoer en gee as afvoer die kleinste heelgetal groter of gelyk aan die getal.

Die vloerfunksie $\lfloor\ \rfloor$ neem 'n reële getal as invoer en gee as afvoer die grootste heelgetal kleiner of gelyk aan die getal.

Dit sal dalk help as jy die getallelyn teken.

Voorbeelde:

- Die plafon van 3.14 is 4 en die vloer van 3.14 is 3:

  $\lfloor 3.14 \rfloor < 3.14 < \lceil 3.14 \rceil$

  Ons kan ook skryf:

  $3 = \lfloor 3.14 \rfloor < 3.14 < \lceil 3.14 \rceil = 4$

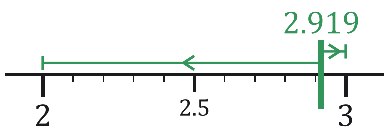
- Die plafon van 2.919 is 3 en die vloer van 2.919 is 2:

  $\lfloor 2.919 \rfloor < 2.919 < \lceil 2.919 \rceil$

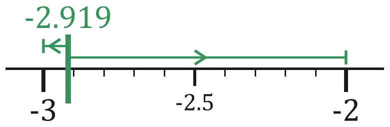
  Ons kan ook skryf:

  $2 = \lfloor 2.919 \rfloor < 2.919 < \lceil 2.919 \rceil = 3$

- Plafon: $\lceil 2.919 \rceil = 3$
  Vloer: $\lfloor 2.919 \rfloor = 2$



- Plafon: $\lceil -2.919 \rceil = -2$
  Vloer: $\lfloor -2.919 \rfloor = -3$



1. $\lfloor 3 \rfloor = \lceil 3 \rceil = 3$
2. $\lceil x \rceil - \lfloor x \rfloor = 0$ as $x \in \mathbb{Z}$
3. $\lceil 7 \rceil - \lfloor 7 \rfloor = 7 - 7 = 0$
4. $\lceil -7 \rceil - \lfloor -7 \rfloor = -7 + 7 = 0$
5. $\lceil x \rceil - \lfloor x \rfloor = 1$ as $x \notin \mathbb{Z}$
6. $\lceil 7{,}32 \rceil - \lfloor 7{,}32 \rfloor = 8 - 7 = 1$
7. $\lceil -7{,}32 \rceil - \lfloor -7{,}32 \rfloor = -7 - (-8) = 1$

5.E Ceiling and Floor

The ceiling function $\lceil \quad \rceil$ takes a real number as input and produces the smallest integer greater than or equal to the number.

The floor function $\lfloor \quad \rfloor$ takes a real number as input and produces the largest integer smaller or equal to the number.

It may help if you draw the number line.

Examples:

- The ceiling of 3.14 is 4 and the floor of 3.14 is 3.

  $\lfloor 3.14 \rfloor < 3.14 < \lceil 3.14 \rceil$

  We can also write:

  $3 = \lfloor 3.14 \rfloor < 3.14 < \lceil 3.14 \rceil = 4$

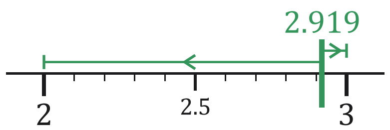
- The ceiling of 2.919 is 3 and the floor of 2.919 is 2.

  $\lfloor 2.919 \rfloor < 2.919 < \lceil 2.919 \rceil$

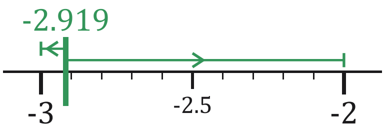
  We can also write:

  $2 = \lfloor 2.919 \rfloor < 2.919 < \lceil 2.919 \rceil = 3$

- ceiling: $\lceil 2.919 \rceil = 3$
  floor: $\lfloor 2.919 \rfloor = 2$



- ceiling: $\lceil -2.919 \rceil = -2$
  floor: $\lfloor -2.919 \rfloor = -3$



1. $\lfloor 3 \rfloor = \lceil 3 \rceil = 3$

2. $\lceil x \rceil - \lfloor x \rfloor = 0$ as $x \in \mathbb{Z}$
3. $\lceil 7 \rceil - \lfloor 7 \rfloor = 7 - 7 = 0$
4. $\lceil -7 \rceil - \lfloor -7 \rfloor = -7 + 7 = 0$

5. $\lceil x \rceil - \lfloor x \rfloor = 1$ as $x \notin \mathbb{Z}$
6. $\lceil 7{,}32 \rceil - \lfloor 7{,}32 \rfloor = 8 - 7 = 1$
7. $\lceil -7{,}32 \rceil - \lfloor -7{,}32 \rfloor = -7 - (-8) = 1$

6.A Geordende pare

As 'n mens die punt P(x; y) op die Cartesiese vlak teken dan maak die volgorde van die twee waardes saak. Die punt P(3; 5) is nie dieselfde as die punt Q(5; 3) nie.

6.E Ordered pairs

If you plot the point P(x; y) on the Cartesian plane the order of the two values matter. The point P(3; 5) is not the same as the point Q(5; 3).

7.A Wat is die laaste syfer(s)?

Die laaste syfer is die enesyfer.

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$, . . .

Die ene-syfers in die antwoord (die *mag*) herhaal: 2, 4, 8, 6. Dis 'n groep van 4 syfers.

As eksponent = 1, dan is 1 ÷ 4 = 0 res 1, Die enesyfer is 2.
As eksponent = 2, dan is 2 ÷ 4 = 0 res 2, Die enesyfer is 4.
As eksponent = 3, dan is 3 ÷ 4 = 0 res 3, Die enesyfer is 8.
As eksponent = 4, dan is 4 ÷ 4 = 1 res 0, Die enesyfer is 6.
As eksponent = 5, dan is 5 ÷ 4 = 1 res 1, . . . Die enesyfer is 2.

★ As ons wil weet wat is die enesyfer van 2^{37} moet ons sê 37 ÷ 4 = *kwosiënt* res 1. Dus eindig 2^{37} op 'n 2.

★ Wat is die enesyfer van 2^{2018} ?
2018 ÷ 4 = … res 2 en as die res 2 is, is die enesyfer 4.

★ Al die magte van 5 eindig in 5; al die magte van 6 eindig in 6.

★ Die laaste twee syfers van die magte van 7 werk in 'n siklus:

$7^1 = 07$
$7^2 = 49$
$7^3 = 3\ 43$
$7^4 = 24\ 01$
$7^5 = 168\ 07$
$7^6 = 1176\ 49$

. . .
07, 49, 43, 01, 07, 49, . . .
Hier is ook groepe van 4.
Eksponent 1 ÷ 4 = 0 res 1 en die laaste twee syfers is 07.
Eksponent 5 ÷ 4 = … res 1 en die laaste twee syfers is 07.

Jy hoef nie elke keer die hele antwoord te kry nie; gebruik net die laaste twee syfers en maal dit met 7.

★ Wat is die laaste twee syfers van 7^{245}?

245 ÷ 4 = … res 1 en as die res 1 is, is die laaste twee syfers 07.

★ Hoe werk hierdie metode vir magte van 3, 4, 7, 8, 9, . . .?

7.E What is the last digit?

The last digit is the ones digit which is also called the units digit.

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$, $2^7 = 128$, . . .

The ones digits in the answer (the *power*) repeat: 2, 4, 8, 6. It is a group of 4 digits.

If exponent = 1, then 1 ÷ 4 = 0 rem 1,
If exponent = 2, then 2 ÷ 4 = 0 rem 2,
If exponent = 3, then 3 ÷ 4 = 0 rem 3,
If exponent = 4, then 4 ÷ 4 = 1 rem 0,
If exponent = 5, then 5 ÷ 4 = 1 rem 1, . . .

★ If we want to know what the units digit of 2^{37} is, we say 37 ÷ 4 = *quotient* rem 1. Hence, 2^{37} ends in a 2.

★ Try 2^{2018} now: 2018 ÷ 4 = … rem 2 and if the rem is 2, the units digit is 4.

★ All the powers of 5 end in 5; all the powers of 6 end in 6.

★ The last two digits of the the powers of 7 also work in a cycle:

$7^1 = 07$
$7^2 = 49$
$7^3 = 3\ 43$
$7^4 = 24\ 01$
$7^5 = 168\ 07$
$7^6 = 1176\ 49$
. . .
07, 49, 43, 01, 07, 49, . . .
Here we also have groups of 4.
Exponent 1 ÷ 4 = 0 rem 1 and the last two digits are 07.
Exponent 5 ÷ 4 = … rem 1 and the last two digits are 07.

You do not need to get the full answer every time; just multiply the last two digits by 7.

★ What are the last two digits of 7^{245}?

245 ÷ 4 = … rem 1 and if the rem is 1, the last two digits are 07.

★ How does this method work for powers of 3, 4, 7, 8, 9, . . .

ALGEMEEN (2) Ken so goed as moontlik vir deelname aan kompetisies.
GENERAL (2) Know as well as possible for participation in competitions.

8. Faktorisering / Factorisation

Gemene faktor / Common factor

(a) $2a + 6b = 2(a + 3b)$

(b) $4x^2 - 8xy = 4x(x - 2y)$

Verskil tussen twee kwadrate / Difference between two squares

(a) $a^2 - b^2 = (a - b)(a + b)$

(b) $\dfrac{1001^2 - 999^2}{101^2 - 99^2} = \dfrac{(1001 - 999)(1001 + 999)}{(101 - 99)(101 + 99)} = 10$

Die volgende voorbeeld sluit 'n gemene faktor in / The next example includes a common factor:

(c) $8x^2 - 18y^4 = 2(4x^2 - 9y^4) = 2(2x - 3y^2)(2x + 3y^2)$

Drieterme / Trinomials

(a) $x^2 + 16x + 63 = (x + 7)(x + 9)$

(b) $x^2 - 17x + 70 = (x - 7)(x - 10)$

(c) $x^2 + 3x - 28 = (x + 7)(x - 4)$

(d) $x^2 - 7x + 12 = (x - 3)(x - 4)$

(e) $2x^2 + 13x + 15 = (2x + 3)(x + 5)$

(f) $3x^2 - 17x + 14 = (3x - 14)(x - 1)$

(g) $5x^2 + 2x - 3 = (5x - 3)(x + 1)$

(h) $5x^2 - 7x - 6 = (5x + 3)(x - 2)$

(i) $5x^2 + 13x - 6 = (5x - 2)(x + 3)$

(j) $4x^2 + 28x - 15 = (2x + 15)(2x - 1)$

Som van derdemagte / Sum of cubes

(1) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(2) $27 + 8c^3 = (3 + 2c)(9 - 6c + 4c^2)$

Verskil tussen derdemagte / Difference between cubes

(1) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(2) $64c^3 - 1 = (4c - 1)(16c^2 + 8c + 1)$

Faktorisering is dikwels nuttig wanneer jy twee groot getalle moet vermenigvuldig:
Factorization is often useful if you have to multiply two big numbers:

★ $299^2 = (300 - 1)^2 = 90000 - 600 + 1 = 89401$

★ $299 \times 298 = (300 - 1)(300 - 2) = 90000 - 600 - 300 + 2 = 89102$

9. Los op vir x / Solve for x

Kry al die terme aan die LK en stel gelyk aan nul. / Get all terms on the LHS and set equal to zero.

(1) Faktoriseer / Factorise

of/or

(2) Kwadratiese formule vir kwadratiese vergelyking / Quadratic formula for quadratic equation

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ALGEMEEN (2) Ken so goed as moontlik vir deelname aan kompetisies.
GENERAL (2) Know as well as possible for participation in competitions.

Voorbeeld / Example

① Faktorisering / ① Factorization

$3x^2 - 17x + 14 = 0$

$(3x - 14)(x - 1) = 0$

$x = 14/3$ of/or $x = 1$

of/or

② Kwadratiese formule / ② Quadratic formula

$3x^2 - 17x + 14 = 0$

a = 3, b = -17, c = 14

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4(3)(14)}}{2(3)}$$

$$x = \frac{17 \pm 11}{6}$$

$$x = \frac{28}{6} = 14/3 = 4\frac{2}{3} \text{ of/or } x = \frac{6}{6} = 1$$

10. Eenvoudige funksienotasie / Simple function notation:

★ y = x + 1 kan geskryf word as/can be written as $f(x) = x + 1$

$f(3.5) = 3.5 + 1 = 4.5$

★ y = 2a – 3 kan geskryf word as/can be written as $f(a) = 2a - 3$

$f(6) = 2(6) - 3 = 12 - 3 = 9$

★ As $f(x) = \frac{3x+1}{x}$, $x \neq 0$ dan is / then

★★ $f(1) = \frac{3(1)+1}{1} = 4$ vervang x met 1/replace x with 1

★★ $f(-x) = \frac{3(-x)+1}{-x} = \frac{-3x+1}{-x} = \frac{3x-1}{x}$ vervang x met $-x$ / replace x with $-x$

★★★ $f(f(x)) = f(\frac{3x+1}{x}) = \frac{3\left(\frac{3x+1}{x}\right)+1}{\frac{3x+1}{x}} = \frac{10x+3}{3x+1}$

Twee stappe: Vervang eers vir $f(x)$ met $\frac{3x+1}{x}$. En vervang dan vir x met die uitdrukking $\frac{3x+1}{x}$ wat tussen hakies is.

Two steps: Replace $f(x)$ with $\frac{3x+1}{x}$. And then replace x with the expression $\frac{3x+1}{x}$ that is between brackets.

★★★ $f(f(1)) = f(\frac{3(1)+1}{1}) = f(4) = \frac{3(4)+1}{4} = \frac{13}{4}$

Hier is ook twee stappe. Probeer dit self.

Here are also two steps. Try it yourself.

11.A Watter getal se faktore? Inleiding.

Hersien gou eers weer 17.A in Algemeen (1).

11.1A Veronderstel jy moet 'n getal vind wat 'n sekere aantal faktore het. Dit is 'n bietjie moeiliker as wanneer jy die getal het en die aantal faktore moet vind.

Die volgende is waar:

- 'n Positiewe heelgetal met 4 verskillende delers is 219.
  - Toets dit: $219 = 3^1 \times 73^1$ en dit gee $2 \times 2 = 4$ faktore.
- 'n Positiewe heelgetal met 20 verskillende faktore is 912.
  - Toets dit: $912 = 2^4 \times 3^1 \times 19^1$ en dit gee $5 \times 2 \times 2 = 20$ faktore.
- Die kleinste positiewe heelgetal met 10 verskillende delers is 48.
- Die kleinste positiewe heelgetal met 15 verskillende delers is 144.
- Vyf positiewe heelgetalle kleiner as 100 wat elk 12 verskillende delers het, is 60, 72, 84, 90 en 96.

11.2A Vind die kleinste positiewe heelgetal met *n* verskillende faktore

★ Vind die kleinste positiewe heelgetal met 5 verskillende faktore:

Die aantal faktore is onewe en dus sal die getal 'n kwadraat (volkome vierkant) wees.

$4 = 2^2$ wat $(2 + 1) = 3$ faktore het.

$9 = 3^2$ wat $(2 + 1) = 3$ faktore het.

$16 = 2^4$ wat $(4 + 1) = 5$ faktore het.

Dit was maklik, want dis 'n klein getal faktore en 'n mens kan dit uitwerk.

★ Vind die kleinste positiewe heelgetal met 12 verskillende faktore.

Begin met die kleinste priemgetal, 2.

2^{11} het 12 faktore, maar is $2^{11} = 2048$.

$2^2 \times 3^3$ het $3 \times 4 = 12$ faktore en is $4 \times 27 = 108$.

$2^2 \times 3 \times 5$ het $3 \times 2 \times 2 = 12$ faktore en is $4 \times 15 = 60$.

Die kleinste getal is 60, wat ook nie te moeilik was om te kry nie.

11.3A Beskou die klompie waardes hieronder. Kan jy enige gevolgtrekking maak?

$2^2 = 4$ het 3 faktore $\quad$ $2^3 = 8$ het 4 faktore $\quad$ $2^4 = 16$ het 5 faktore

$2^5 = 32$ het 6 faktore	$2^6 = 64$ het 7 faktore	$2^7 = 128$ het 8 faktore
$2^8 = 256$ het 9 faktore	$2^9 = 512$ het 10 faktore	
$3^2 = 9$ het 3 faktore	$3^3 = 27$ het 4 faktore	$3^4 = 81$ het 5 faktore
$3^5 = 243$ het 6 faktore	$3^6 = 729$ het 7 faktore	$3^7 = 2187$ het 8 faktore
$3^8 = 6561$ het 9 faktore	$3^9 = 19683$ het 10 faktore	

$2^1 \times 3^1 = 6$ het 4 faktore

$2^2 \times 3^1 = 12$ het 6 faktore

$2^1 \times 3^2 = 18$ het 6 faktore

$2^3 \times 3^1 = 24$ het 8 faktore

$2^1 \times 3^3 = 54$ het 8 faktore

11.E Which number's factors? Introduction.

First, revise 17.E in General (1).

11.1E Suppose you have to find a number that has a specific number of factors. This is a bit more difficult than to find the number of factors if you have the number.

The following is true:

- A positive integer with 4 distinct (different) factors (divisors) is 219.
  - Test it: $219 = 3^1 \times 73^1$ which gives $2 \times 2 = 4$ factors.
- A positive integer with 20 distinct divisors is 912.
  - Test it: $912 = 2^4 \times 3^1 \times 19^1$ which gives $5 \times 2 \times 2 = 20$ factors.
- The smallest positive integer with 10 distinct divisors is 48.
- The smallest positive integer with 15 distinct divisors is 144.
- Five positive integers less than 100 with 12 distinct divisors each, are 60, 72, 84, 90 and 96.

11.2E Find the smallest positive integer with *n* distinct factors.

★ Find the smallest positive integer with 5 distinct factors:

The number of factors is odd and therefore the number is perfect square.

$4 = 2^2$ has $(2 + 1) = 3$ factors.

$9 = 3^2$ has $(2 + 1) = 3$ factors.

$16 = 2^4$ has $(4 + 1) = 5$ factors.

This was easy, since we worked with a small number of factors and you can work it out.

★ Find the smallest positive integer with 12 distinct factors.

Start with the smallest prime number, 2.

2^{11} has 12 factors, but $2^{11} = 2048$.

$2^2 \times 3^3$ has $3 \times 4 = 12$ factors and is equal to $4 \times 27 = 108$.

$2^2 \times 3 \times 5$ has $3 \times 2 \times 2 = 12$ factors and is equal to $4 \times 15 = 60$.

The smallest number is 60, which was not too difficult to obtain.

11.3E Consider the following values. Can you draw any conclusions?

ALGEMEEN (2) Ken so goed as moontlik vir deelname aan kompetisies.
GENERAL (2) Know as well as possible for participation in competitions.

$2^2 = 4$ has 3 factors
$2^3 = 8$ has 4 factors
$2^4 = 16$ has 5 factors
$2^5 = 32$ has 6 factors
$2^6 = 64$ has 7 factors
$2^7 = 128$ has 8 factors
$2^8 = 256$ has 9 factors
$2^9 = 512$ has 10 factors

$3^2 = 9$ has 3 factors
$3^3 = 27$ has 4 factors
$3^4 = 81$ has 5 factors
$3^5 = 243$ has 6 factors
$3^6 = 729$ has 7 factors
$3^7 = 2187$ has 8 factors
$3^8 = 6561$ has 9 factors
$3^9 = 19683$ has 10 factors

$2^1 \times 3^1 = 6$ has 4 factors

$2^2 \times 3^1 = 12$ has 6 factors

$2^1 \times 3^2 = 18$ has 6 factors

$2^3 \times 3^1 = 24$ has 8 factors

$2^1 \times 3^3 = 54$ has 8 factors

12.A Waatlemoen

'n Baie groot waatlemoen wat 100 kg weeg, bestaan uit 99% water. Nadat dit vir 'n paar uur in die son gelê het, bevat dit 98% water. Wat weeg die waatlemoen nou?

Stem jy saam dat die waatlemoen nou slegs 50 kg weeg?

Die belangrike feit hier is dat die waatlemoen uit water en vaste stof bestaan. Die vaste stof kan nie verdamp nie en dit is dieselfde massa voor en na die verdamping van die water in die waatlemoen.

12.E Watermelon

A very big watermelon of 100 kg contains 99% water. After standing in the sun for a few hours, it contains 98% water. How much does the watermelon weigh now?

Do you agree that the watermelon now weighs only 50 kg?

The important thing to keep in mind here is that the watermelon consists of water and a solid part. The solid part cannot evaporate and is the same mass before and after the evaporation of the water in the watermelon.

https://thatsmaths.com/2013/11/14/the-watermelon-puzzle/

Einde van Algemeen (2) / End of General (2)