ALGEMEEN 3 / GENERAL 3 Gr 7 - 9

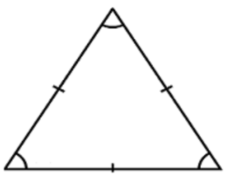
MEETKUNDE / GEOMETRY

1. Driehoeke / Triangles

1. Die som van die binnehoeke van 'n driehoek is 180°.
2. Die som van die lengtes van enige twee sye van 'n driehoek is groter as die derde sy.
3. Al die hoeke van 'n skerp(hoekige) driehoek is kleiner as 90°.
4. Een hoek van 'n stomphoekige driehoek is groter as 90°.
5. Een hoek van 'n reghoekige driehoek is gelyk aan 90°.
6. 'n Ongelyksydige driehoek het geen gelyke sye of hoeke nie.
7. Die buitehoek van 'n driehoek is gelyk aan die som van die twee teenoorstaande binnehoeke.

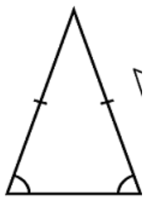
1. The sum of the interior angles of a triangle is 180°.
2. The sum of the lengths of any two sides of a triangle is greater than the third side.
3. All angles of an acute triangle are less than 90°.
4. One angle of an obtuse triangle is greater than 90°.
5. One angle of a right-angled triangle is equal to 90°.
6. A scalene triangle has no equal sides or angles.
7. The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

Gelyksydige Δ / Equilateral Δ



Al drie sye is ewe lank. Al drie hoeke is ewe groot. Elke hoek is 60°.	All three sides are equal. All three angles are equal. Each angle is 60°.

Gelykbenige Δ / Isosceles Δ



Twee sye is ewe lank. Die twee hoeke oorstaande aan die gelyke sye is ewe groot.	Two sides are equal. The two angles opposite the two equal sides are equal.

1. Oppervlakte van 'n driehoek = ½ × basis × hoogte = $\frac{1}{2}b \cdot h$ = $\frac{1}{2}h \cdot b$: gebruik dit wanneer die hoogte op hierdie basis bekend is.

of

2. Oppervlakte van 'n driehoek = $\sqrt{s(s-a)(s-b)(s-c)}$ waar s = ½ × omtrek en a, b en c die sylengtes is (Heron se formule): gebruik dit wanneer die drie sye bekend is.

of

X. Oppervlakte van 'n driehoek = $\frac{1}{2}a \cdot b \cdot \sin C$ (formule met trigonometrie): gebruik dit wanneer twee sye en die ingeslote hoek bekend is (bedoel vir gr 10 – 12).

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1. Area of a triangle = ½ × base × height = $\frac{1}{2}b \cdot h = \frac{1}{2}h \cdot b$: use it when the height on this base is known.

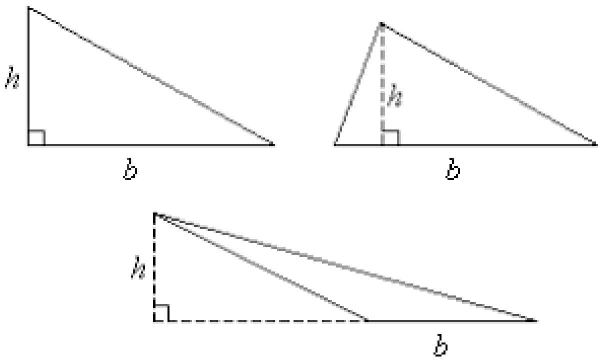
or

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where s = ½ × perimeter and a, b and c the sides (Heron's formula): use it when the three sides are known.

or

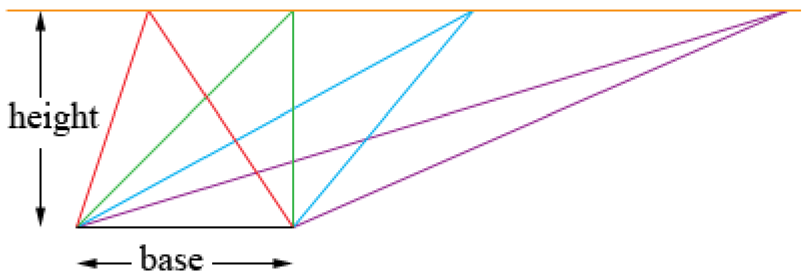
X. Area of a triangle = $\frac{1}{2}a \cdot b \cdot \sin C$ (formula with trigonometry): use it when two sides and the included angle are known (meant for gr 10 – 12).

Vir 'n reghoekige Δ is die hoogte op 'n reg-hoeksy as die basis 'n reghoeksy is.	For a right-angled Δ the height is on a right-angled side if the base is a rightangled side.
Vir 'n skerp Δ is die hoogte binne die Δ.	For an acute Δ the height is inside the Δ.
Vir 'n stomp Δ is die hoogte "buite" die Δ; op die verlengde van die basis.	For an obtuse Δ the height is "outside" the Δ, i.e. on the extension of the base.
Sien die figure hieronder.	See the figures below.



Driehoeke met dieselfde (of gelyke) basisse en dieselfde (of gelyke) hoogtes het gelyke oppervlaktes.

Triangles with the same (or equal) bases and the same (or equal) heights have equal areas.



2.1 Vierhoeke / Quadrilaterals

Vierhoek (poligoon met vier sye) / Quadrilateral (four-sided polygon)

2-dimensioneel 2-dimensional

★ Vierkant: Vier gelyke sye; oorstaande sye is ewewydig; elke hoek is 90°.
★ Square: Four equal sides; opposite sides are parallel; each angle is 90°.

★ *Reghoek*: Albei pare oorstaande sye is gelyk en ewewydig; elke hoek is 90°.
★ *Rectange*: Both pairs of opposite sides are equal and parallel; each angle is 90°.

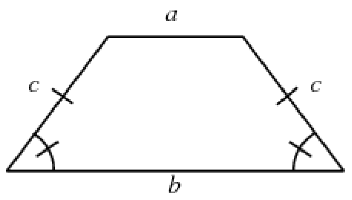
★ Parallelogram: Albei pare oorstaande sye is gelyk en ewewydig, maar hoeke is nie 90° nie. As jy 'n parallelogram se hoek almal 90° maak, is dit 'n reghoek.
★ Parallelogram: Both pairs of opposite sides are equal and parallel, but angles are not 90°. If you make all the angles of a parallelogram 90° it will be a rectangle.

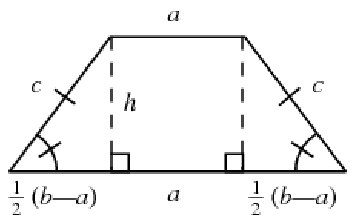
★ *Ruit*: Albei pare oorstaande sye is gelyk en ewewydig, maar die hoeke is nie 90° nie.
Hoeklyne van 'n ruit is loodreg op mekaar
★ *Rhombus*: Both pairs of opposite sides are equal and parallel, but the the angles are not 90°.
Diagonals of a rhombus are perpendicular to each other

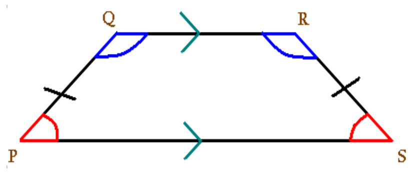
★ Trapesium: Een paar oorstaande sye is ewewydig.
Sy hoogte is die afstand tussen die ewewydige sye.

★ Trapezium: One pair of opposite sides are parallel.
Its height is the distance between the parallel sides.

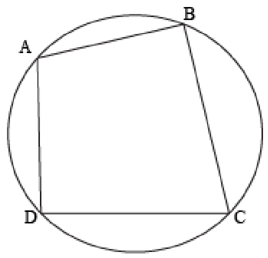
★ Gelykbenige trapesium / Isosceles trapezium



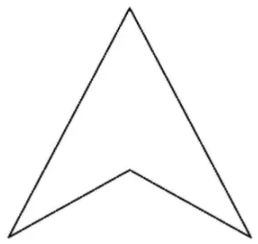




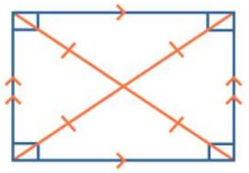
★ *Koordevierhoek*: 'n Koordevierhoek se hoekpunte is al vier op 'n sirkel se omtrek.
★ *Cyclic quadrilateral*: A cyclic quadrilateral's vertices are all on the circumference of a circle.



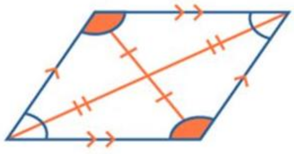
★ *Pylpunt*: 2 gelyk kort sye en 2 gelyke lang sye; die 2 skerp hoeke ingesluit tussen die lang en kort sye is gelyk.
★ *Arrowhead*: 2 equal short sides and 2 equal longer sides; the 2 acute angles included between the long and short sides are equal.



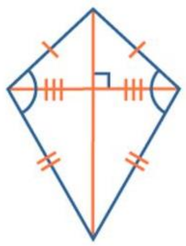
2.2 Hoeklyne van vierhoeke / Diagonals of quadrilaterals



Reghoek	Rectangle
Hoeklyne is ewe lank.	Diagonals have the same length.
Hoeklyne is nie loodreg op mekaar nie.	Diagonals are not perpendicular to each other.
Hoeklyne halveer mekaar.	Diagonals bisect each other.



Parallelogram	Parallelogram
Hoeklyne is nie ewe lank nie.	Diagonals have different lengths.
Hoeklyne is nie loodreg op mekaar nie.	Diagonals are not perpendicular to each other.
Hoeklyne halveer mekaar.	Diagonals bisect each other.



Vlieër	Kite
Hoeklyne is nie ewe lank nie.	Diagonals have different lengths.
Hoeklyne is loodreg op mekaar.	Diagonals are perpendicular to each other.
Een hoeklyne word gehalveer.	One diagonals is bisected.

2.3 Konveks / Convex

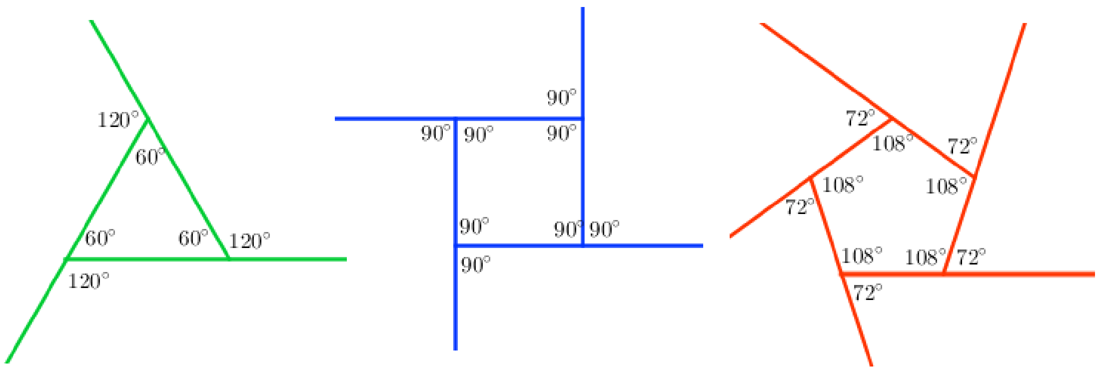
Voorbeelde: As jy al die hoekpunte van 'n veelhoek twee-twee met mekaar kan verbind sonder dat enige van die verbindingslyne buite die veelhoek is, dan is die veelhoek konveks, bv. driehoek, parallelogram, trapesium. Die pylpunt is nie konveks nie; dis konkaaf.

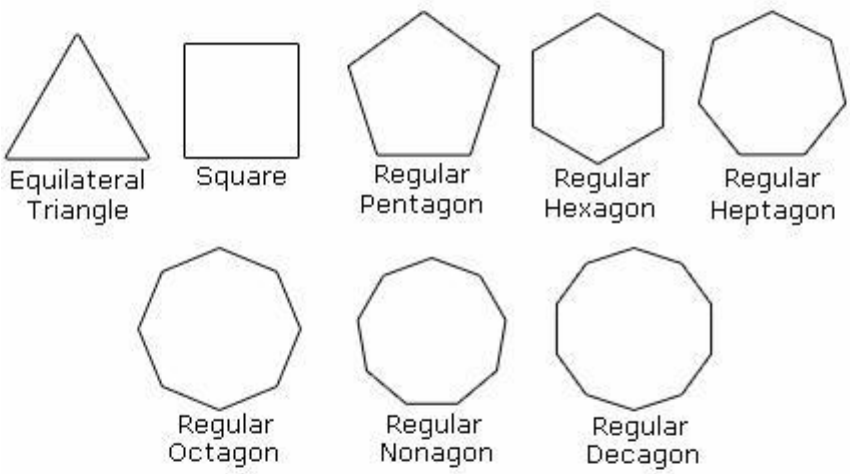
Examples: If you can join all the vertices of a polygon in pairs and the lines are on or inside the polygon then the polygon is convex, e.g. triangle, parallelogram, trapezium. The arrowhead is not convex; it is concave.

2.4 Reëlmatig / Regular

Voorbeelde: 'n Seshoek waarvan al 6 sye ewe lank is en al 6 hoeke ewe groot is, is reëlmatig. Dieselfde geld vir 'n vierkant, 'n gelyksydige driehoek en vir ander vorms met gelyke sye en ewe groot hoeke.

Examples: A hexagon with 6 equal sides and 6 equal angles is regular. This is also true for a square, an equilateral triangle and other shapes with equal sides and equal angles.



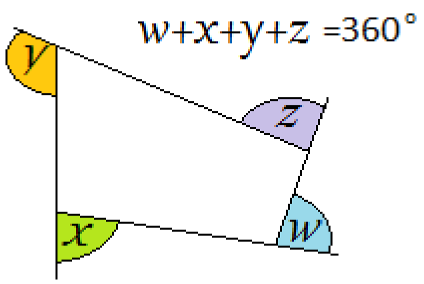


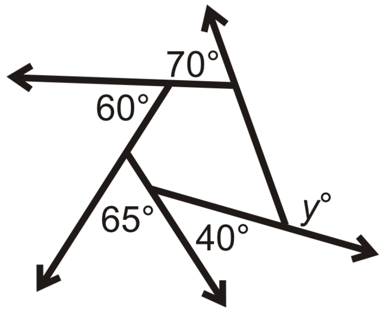
Onreëlmatige veelhoeke: die buitehoeke is nie almal ewe groot nie, maar die som van die buitehoeke is steeds 360°.

Sommige binnehoeke kan dieselfde groottes hê en ander kan verskil; netso vir die sye. Of, al die binnehoeke kan verskillende groottes hê en al die sye kan verskillende lengtes hê.

Irregular polygons: the exterior angles are not all equal, but the sum of the exterior angles are still equal to 360°.

The interior angles can all have different sizes or some can be equal and other not; the sides can all have different lengths, or some can have equal lengths and some not.





2.5 Oppervlakte / Area

- Oppervlakte van 'n vierkant = sy × sy
- Area of a square = side × side

- Oppervlakte van 'n reghoek = lengte × breedte
- Area of a rectangle = length × width

- Oppervlakte van 'n parallelogram = basis × hoogte op daardie basis
- Area of a parallelogram = base × height on that base

- Oppervlakte van 'n ruit = basis × hoogte op daardie basis
- Area of a rhombus = base × height on that base

- Oppervlakte van 'n ruit = $\frac{1}{2}$(produk van die hoeklyne)

- Area of a rhombus = $\frac{1}{2}$(product of diagonals)

- Oppervlakte van 'n trapesium = $\frac{1}{2}$(som van die ewewydige sye) × hoogte

- Area of a trapezium = $\frac{1}{2}$(sum of parallel sides) × height

- Oppervlakte van 'n koordevierhoek = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

  en s = $\frac{a+b+c+d}{2}$ (Brahmagupta se formule)

- Area of a cyclic quadrilateral = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$

  and s = $\frac{a+b+c+d}{2}$ (Brahmagupta's formula)

2.6 Middellyne, middelpunte / Bisectors, midpoints

- In die middel van 'n sirkel het ons sy middelpunt.
- In the middle of a circle we have its centre.

https://www.youtube.com/watch?v=1oJTBwnZYVw (2:44)

- Die lyn wat 'n hoek in twee gelyke dele verdeel, is 'n halveerlyn.
- The line that divides an angle into two equal parts is a bisector.
- Die lyn wat 'n lynstuk in twee gelyke dele verdeel, is 'n halveerlyn. In die middel van 'n lynstuk het ons sy middelpunt.
- The line that divides a line segment into two equal parts is a bisector. In the middle of a line segment we have its midpoint.

https://www.youtube.com/watch?reload=9&v=lIwvSzfUkOY (2:15)

- Loodregte halveerlyn van 'n lynsegment
- Perpendicular bisector of a line segment.

https://www.youtube.com/watch?v=5bvjnleMn5A (1:38)

2.7 Konstruksies: ken / Constructions: know

- Vier konstruksies / Four constructions

  https://www.youtube.com/watch?v=wDJrOWMeYOc (14:12)

- Om 'n vierkant binne 'n gegewe sirkel te teken / To draw a square inside a given circle

  https://www.youtube.com/watch?v=2gNfltBkbkI (2:25)

- X Om 'n reëlmatige seshoek binne 'n gegewe sirkel te teken / To draw a square inside a given circle

  https://www.youtube.com/watch?v=USq_4tthP5Q (3:15)

- X Om 'n reëlmatige vyfhoek binne 'n gegewe sirkel te teken / To draw a square inside a given circle

  https://www.youtube.com/watch?v=9NmO1Bq-oWg (5:32)

  https://www.youtube.com/watch?v=aB78NVU3u0Y (4:19)
  https://www.youtube.com/watch?v=0I-Ly8g3IIE (1:40)

- X Om 'n pentagram (vyfpuntster) te konstrueer / To construct a pentagram (five-pointed star)

  https://www.youtube.com/watch?v=pQn3ZTVSGu4 (7:39)

  https://www.youtube.com/watch?v=qpYQZGczhDk (3:08) (vyfhoek gegee / pentagon given)

2.8 Die som van die binnehoeke van ‘n veelhoek word gegee deur
$180° \times (n - 2)$ waar n die aantal sye van die veelhoek is; $n \geq 3$.

The sum of the interior angles of a polygon is given by
$180° \times (n - 2)$ where n is the number of sides of the polygon; $n \geq 3$.

2.9 Die som van die buitehoeke van enige konvekse veelhoek is 360°:

$n \times 180° - (n - 2) \times 180° = 2 \times 180°$ waar n = aantal sye en $n \geq 3$.

The sum of the exterior angles of any convex polygon is 360°:

$$n \times 180° - (n-2) \times 180° = 2 \times 180° \text{ where } n = \text{number of sides and } n \geq 3.$$

3. Sirkels / Circles

3.1 'n Sirkel het 'n radius / A circle has a radius

Die middellyn (d) van 'n sirkel = 2 keer sy radius.
The diameter (d) of a circle = 2 × its radius.

$$d = 2r$$

$$\pi = \frac{\text{omtrek van die sirkel}}{\text{middellyn}} \approx 3.14 \qquad \pi = \frac{\text{circumference of the circle}}{\text{diameter}} \approx 3.14$$

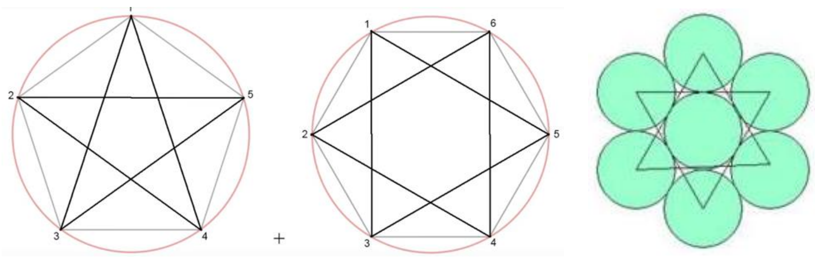
3.14 and $^{22}/_{7}$ is rasionale benaderings vir π, want π is eintlik 'n irrasionale getal.

3.14 and $^{22}/_{7}$ are rational approximations of π, since π is actually an irrational number.

Oppervlakte van 'n sirkel = πr^2 = area of a circle

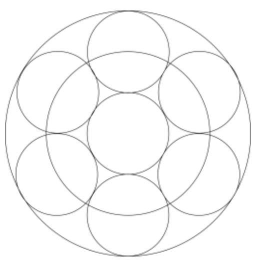
Omtrek van 'n sirkel = $2\pi r$ = circumference of a circle

3.2 Links onder is die vyfpuntster en regs is die Dawidster.
On the left below is the five-pointed star and on the right is the Star of David.



3.3 Hieronder: Die sewe klein sirkels het almal dieselfde radius, naamlik r. Die ses klein sirkels wat aan die binneste klein sirkel raak, pas presies in rondom die binneste klein sirkel. Die medium sirkel (wat deur die middelpunte van die ses klein sirkels gaan) se radius is $2r$ en die groot sirkel se radius is $3r$.

Below: The seven small circles all have the same radius, i.e. r. The six small circles that touch the small inner circle, fit exactly around the small inner circle. The medium circle (passing through the centres of the six small circles) has radius $2r$ and the large circle has radius $3r$.



3.4A Verhouding (sirkel)

Radius van sirkel $O_1 = 5 = r_1$
Radius van sirkel $O_2 = 15 = r_2$

Oppv van sirkel $O_1 = 25\pi = A_1$
Oppv van sirkel $O_2 = 225\pi = A_2$

Verhoudings:

$r_1 : r_2 = 5 : 15 = 1 : 3$

$A_1 : A_2 = 25\pi : 225\pi = 1 : 9$

Die verhouding van die oppv van sirkels is die kwadrate van die verhoudings van die onderskeie radii.

Halwe sirkels?

3.4E Ratio (circle)

Radius of circle $O_1 = 5 = r_1$
Radius of circle $O_2 = 15 = r_2$

Area of circle $O_1 = 25\pi = A_1$
Area of circle $O_2 = 225\pi = A_2$

Ratios:

$r_1 : r_2 = 5 : 15 = 1 : 3$

$A_1 : A_2 = 25\pi : 225\pi = 1 : 9$

The ratio of the area of circles are the squares of the ratios of the respective radii.

Semi-circles?

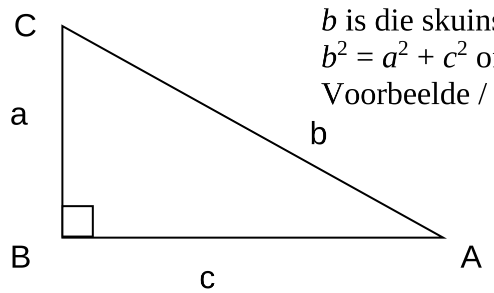
4. PYTHAGORAS ('n Griekse wiskundige / A Greek mathematician)

Vir enige driehoek geld dat die som van die lengtes van enige twee sye groter is as die derde sy.

For any triangle it is true that the sum of the lengths of any two sides will be larger than the third side.

In 'n reghoekige driehoek is die vierkant op die skuinssy gelyk aan die som van die vierkante op die twee reghoeksye. 'n Reghoekige driehoek het een hoek wat gelyk is aan 90°. Die skuinssy is altyd die langste sy.

In a right-angled triangle (right triangle), the square on the hypotenuse is equal to the sum of the squares on the right-angled sides (legs). A right-angled triangle has one angle of 90°. The hypotenuse is always the longest side.

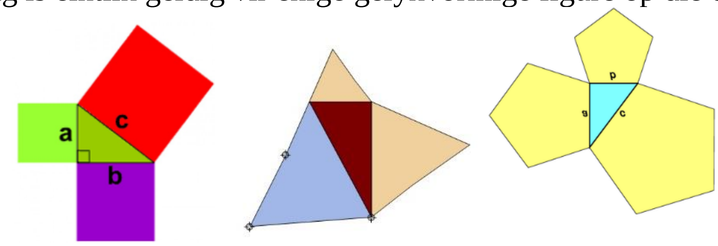


b is die skuinssy / *b* is the hypotenuse
$b^2 = a^2 + c^2$ of/*or* $AC^2 = BC^2 + AB^2$
Voorbeelde / Examples:

$5^2 = 4^2 + 3^2$
$13^2 = 12^2 + 5^2$
(ook veelvoude, bv. $15^2 = 12^2 + 9^2$;
also multiples, e.g. $15^2 = 12^2 + 9^2$)

Sulke getalle word Pythagoras-drietalle genoem. / Such numbers are called Pythagorean triples.

This theorem is actually valid for any similar figures on the three sides.
Hierdie stelling is eintlik geldig vir enige gelykvormige figure op die drie sye.



5. Ewewydige lyne / Parallel lines

https://www.mathsisfun.com/geometry/parallel-lines.html

terminologie / terminology

ooreenstemmende hoeke = corresponding angles

verwisselende hoeke = alternate angles

snylyn = transversal

regoorstaande hoeke = vertically opposite angles

verwisselende binnehoeke = alternate interior angles

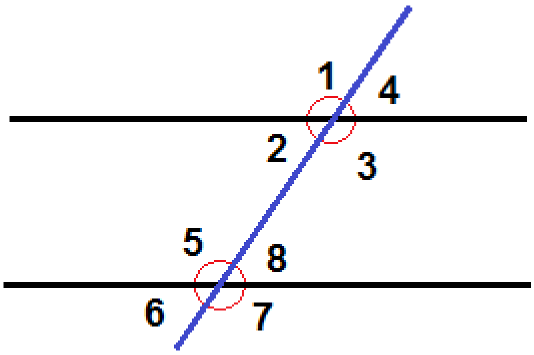
verwisselende buitehoeke = alternate exterior angles

ko-binnehoeke = co-interior angles =

binnehoeke aan dieselfde kant van die snylyn = interior angles on the same side of the transversal (consecutive interior angles)

Sommige hoeke met hul verwantskappe word hieronder gegee. Die twee swart lyne in die figuur is ewewydig.

Some angles with their relationships are given below. The two black lines in the figure are parallel.



$\angle 1 = \angle 3$…….regoorstaande/vertically opposite
$\angle 2 = \angle 8$…….verwisselend/alternate
$\angle 1 = \angle 7$…….verwisselend/alternate
$\angle 5 = \angle 3$…….verwisselend/alternate
$\angle 2 + \angle 5 = 180°$……ko-binnehoeke/co-interior angles
$\angle 3 + \angle 8 = 180°$…… ko-binnehoeke/co-interior angles
$\angle 4 = \angle 6$…….verwisselende buitehoeke/alternate exterior angles
$\angle 4 = \angle 8$…….ooreenstemmende hoeke/corresponding angles
$\angle 2 + \angle 3 = 180°$…. gestrekte hoek/straight angle

6. Supplementêr en Komplementêr / Supplementary and Complementary

Twee komplementêre hoeke is saam 90°; hulle hoef nie aanliggend te wees nie..
Twee supplementêre hoeke is saam 180°; hulle hoef nie aanliggend te wees nie.

Two complementary angles add up to 90°; they need not be adjacent.
Two supplementary angles add up to 180°; they need not be adjacent.

As twee reguitlyne mekaar sny, is die regoorstaande hoeke ewe groot.
If two straight lines intersect, the vertically opposite angles are equal.

7. Kongruensie van driehoeke / Congruency of triangles

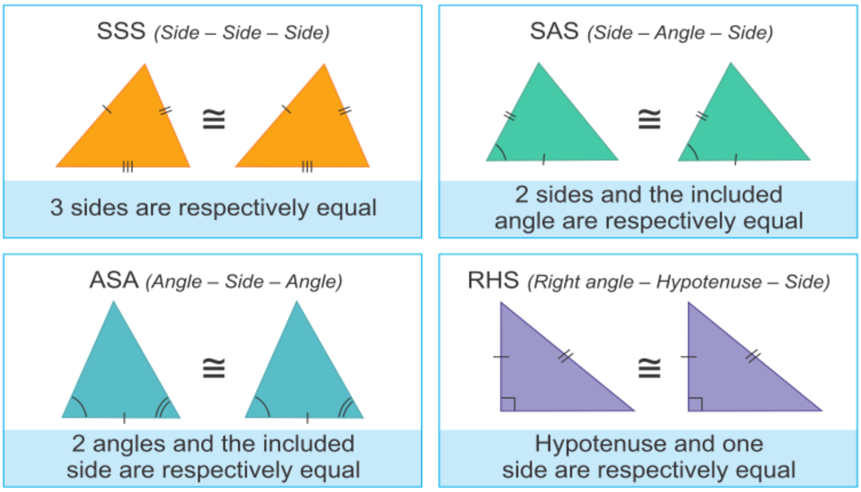
As twee of meer figure dieselfde vorm en grootte het, is hulle kongruent.
If two or more figures have the same shape and size, they are congruent.

Kongruensie van driehoeke

1) Die drie sye van een driehoek is gelyk aan die ooreenstemmende sye van die ander driehoek.
2) Twee hoeke en een sy van 'n driehoek is gelyk aan twee hoeke en die ooreenstemmende sy van die ander driehoek.
3) Twee sye en die ingeslote hoek van een driehoek is gelyk aan twee sye en die ingeslote hoek van die ander driehoek.
4) Die skuinssy en een reghoeksy van 'n reghoekige driehoek is respektiewelik gelyk aan die skuinssy en een reghoeksy van die ander driehoek.

Congruency of triangles

1) The three sides of one triangle are equal to the corresponding sides of the other triangle.
2) Two angles and one side of a triangle are equal to two angles and the corresponding side of the other triangle.
3) Two sides and the included angle of one triangle are equal to two sides and the included triangle of the other triangle.
4) The hypotenuse and one right-angled side of a right-angled triangle are equal to the hypotenuse and one right-angled side of the other right-angled triangle.



Daar is dus vier gevalle waar driehoeke kongruent sal wees.

There are therefore four cases where triangles will be congruent.

Notasie: ΔABC ≡ ΔPQR
(Skryf die gelyke dele in die ooreenstemmende volgorde neer.)
Dit beteken in hierdie geval dat AB = PQ, AC = PR, BC = QR, $\hat{A}=\hat{P}$, $\hat{B}=\hat{Q}$, $\hat{C}=\hat{R}$.

Notation: ΔABC ≡ ΔPQR
(Write the equal parts in the corresponding order.)
It means that in this case we have AB = PQ, AC = PR, BC = QR, $\hat{A}=\hat{P}$, $\hat{B}=\hat{Q}$, $\hat{C}=\hat{R}$.

8.A Gelykvormigheid

Twee veelhoeke (bv. driehoeke) is gelykvormig as al die pare ooreenstemmende hoeke gelyk is en as al die pare ooreenstemmende sye in dieselfde verhouding is.

Die omgekeerde geld ook: As veelhoeke gelykvormig is, is al die pare ooreenstemmende hoeke gelyk en al die pare ooreenstemmende sye in dieselfde verhouding.

* Kongruente driehoeke is gelykvormig.
* Gelykvormige driehoeke is nie noodwendig kongruent nie.

Notasie:
ΔABC ||| ΔPQR of soms ΔABC ~ ΔPQR

(Skryf die gelyke hoeke in die ooreenstemmende volgorde neer.)

Dit beteken dat ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R en dat $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$.

By driehoeke is slegs die gelyke hoeke **of** die eweredigheid van die sye nodig.

8.E Similarity

Two polygons (e.g. triangles) are similar if all the corresponding pairs of angles are equal and if all the corresponding sides are in the same ratio.

The converse is also true: If polygons are similar all the pairs of angles are equal and all the corresponding sides are in the same ratio.

* Congruent triangles are similar.
* Similar triangles are not necessarily congruent.

Notation:
ΔABC ||| ΔPQR or sometimes ΔABC ~ ΔPQR
(Write the equal angles in the corresponding positions.)
That means that ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R and that $\frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$.

With triangles it is only necessary to have either the equal angles **or** the ratio of the sides.

9. Volume

3D-vorms / 3D shapes

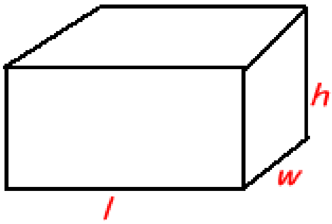
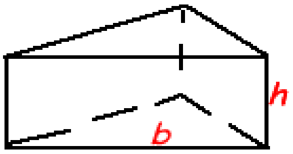
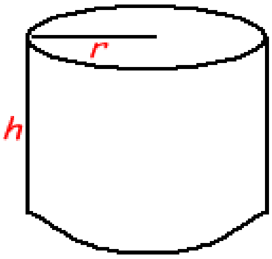
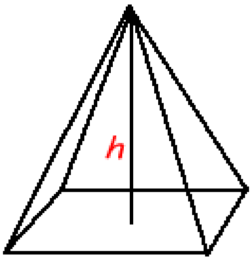
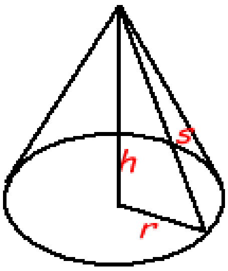
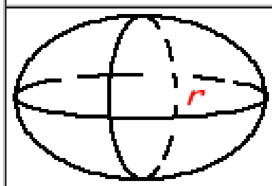
Ken die formules vir die volumes van die mees algemene vorms.
Know the formulas for the volumes of the most common shapes.

https://www.science.co.il/formula/

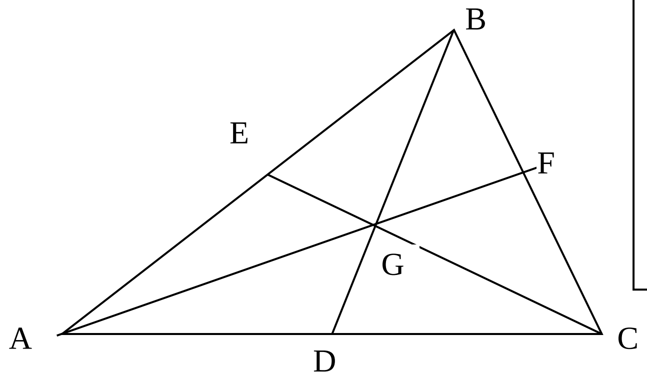
http://onlinemschool.com/math/formula/volume/

https://www.thoughtco.com/surface-area-and-volume-formulas-604148

rectangular solid = reghoekige blok		
prism = prisma	cylinder = silinder	
pyramid = piramide	cone = keël	sphere = sfeer
faces = vlakke	edges = kante / rande	vertices = hoekpunte

	**Rectangular Solid** Volume = Length X Width X Height V = lwh Surface = 2lw + 2lh + 2wh
	**Prisms** Volume = Base X Height v=bh Surface = 2b + Ph *(b is the area of the base P is the perimeter of the base)*
	**Cylinder** Volume = πr^2 x height V = πr^2 h Surface = 2π radius x height S = $2\pi rh + 2\pi r^2$
	**Pyramid** V = 1/3 bh b is the area of the base Surface Area: Add the area of the base to the sum of the areas of all of the triangular faces. The areas of the triangular faces will have different formulas for different shaped bases.
	**Cones** Volume = 1/3 πr^2 x height V= 1/3 $\pi r^2 h$ Surface = $\pi r^2 + \pi rs$ S = $\pi r^2 + \pi rs$ $=\pi r^2 + \pi r\sqrt{r^2 + h^2}$
	**Sphere** Volume = 4/3 πr^3 V = 4/3 πr^3 Surface = $4\pi r^2$ S = $4\pi r^2$

10. Mediane in driehoeke / Medians in triangles



'n Mediaan gaan van 'n hoekpunt na die middelpunt van die oorstaande sy.

A median is drawn from a vertex to the midpoint of the opposite side.

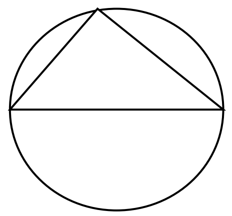
EC, AF en BD is die mediane.
E, D en F die middelpunte
van AB, AC en BC onderskeidelik.

EC, AF and BD are the medians.
E, D and F are the midpoints
of AB, AC and BC respectively.

$$AG = \frac{2}{3}AF,\ \ BG = \frac{2}{3}BD,\ \ CG = \frac{2}{3}CE$$

11. Trek 'n sirkel en trek die middellyn. Die middellyn gaan deur die middelpunt van die sirkel. Voltooi 'n driehoek. Die hoek op die sirkel is gelyk aan 90°.

Draw a circle and draw the diameter. The diameter passes through the centre of the circle. Complete a triangle. The angle on the circle is equal to 90°.



12. Die omtrek van die ingeskrewe reëlmatige seshoek in 'n sirkel is drie keer die middellyn van die sirkel.

The perimeter of the inscribed regular hexagon in a circle is three times the diameter of the circle.

http://mathcentral.uregina.ca/qq/database/qq.09.07/s/vivek1.html

13.A Skaalfaktor

As die sylengte van 'n vierkant 1 cm is en die sylengte van 'n ander vierkant 2 cm, dan is die skaalfaktor 1 : 2. Let daarop dat vierkante gelykvormig is.

As die omtrek van een vierkant 5 cm is, is die omtrek van die ander vierkant 10 cm indien die skaalfaktor 1 : 2 is.

As die oppv van een vierkant 25 cm^2 is, is die oppv van die ander vierkant 25×3^2 cm^2 = 225 cm^2 indien die skaalfaktor 1 : 3 is.

As die volume van een kubus 9 cm^3 is, is die volume van die ander kubus 9×2^3 cm^2 = 72 cm^3 indien die skaalfaktor 1 : 2 is.

As die skaalfaktor vir gelykvormige figure $a : b$ is, is die verhouding van die oppervlaktes van daardie figure $a^2 : b^2$.

Voorbeeld: Die sye van vierhoek A is 7, 8, 6 en 4 cm. Die oppervlakte is 56 cm^2. Wat is die oppervlakte van vierhoek B, gelykvormig aan A, met langste sy 12 cm?
skaalfaktor is $A : B = 8 : 12$
en $\text{oppv}_A : \text{oppv}_B = 64 : 144$

Dus: $\frac{56}{x} = \frac{64}{144}$ en $x = 126$ cm^2.

13.E Scale factor

If the sidelength of a square is 1 cm and the sidelength of another square is 2 cm, then the scale factor is 1 : 2. Note that squares are similar.

If the perimeter of a square is 5 cm, then the perimeter of the other square is 10 cm if the scale factor is 1 : 2.

If the area of a square is 25 cm^2, then the area of the other square is 25×3^2 cm^2 = 225 cm^2 if the scale factor is 1 : 3.

If the volume of a cube is 9 cm^3, then the volume of the other cube is 9×2^3 cm^2 = 72 cm^3 if the scale factor is 1 : 2.

If the scale factor of similar figures is a:b, then the ratio of the areas of those figures is $a^2 : b^2$.

Example: The sides of quadrilateral A are 7, 8, 6 and 4 cm. The area is 56 cm^2. The area of a similar quadrilateral B, with longest side 12 cm is:

scale factor is $A : B = 8 : 12$
and $\text{area}_A : \text{area}_B = 64 : 144$

$\therefore \frac{56}{x} = \frac{64}{144}$ and $x = 126$ cm^2.

14. Transformasiemeetkunde / Transformation Geometry

Die Cartesiese vlak is vernoem na die wiskundige Rene Descartes (1596 - 1650). Dit is ‘n vlak met ‘n reghoekige koördinaatstelsel wat elke punt in die vlak met een paar getalle assosieer.

The Cartesian plane is named after the mathematician Rene Descartes (1596 - 1650). It is a plane with a rectangular coordinate system that associates each point in the plane with one pair of numbers.

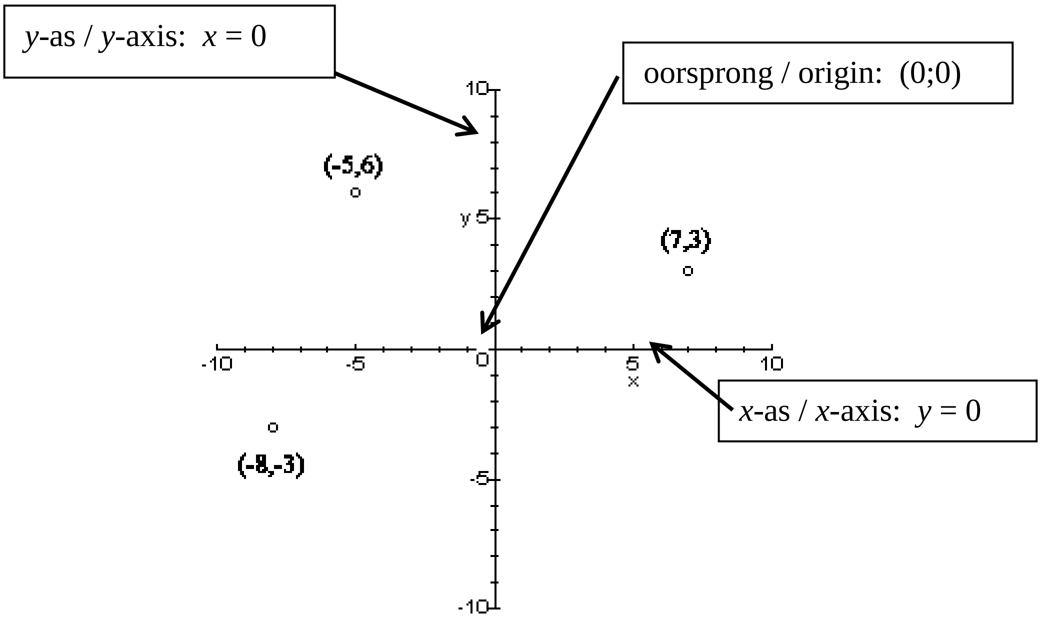
Die ligging (posisie) van ‘n punt P word bepaal deur die geordende getallepaar $(x;y)$, ook genoem P se koördinate.

The location (position) of a point P is determined by an ordered number pair $(x;y)$, also called the coordinates of P.

14.1 Die afstand tussen twee punte $A(x_1, y_1)$ en $B(x_2, y_2)$ op die Cartesiese vlak (gebruik die stelling van Pythagoras om hierdie formule te vind) is

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ on the Cartesian plane (use the theorem of Pythagoras to find this formula):

$$AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



14.2 Gradiënt van 'n reguit lyn / Gradient of a straight line:

$m_{AB} = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y_2 - y_1}{x_2 - x_1}$ waar / *where* A(x_1, y_1) en / *and* B(x_2, y_2) twee punte op die lyn is / are two points on the line.

- Twee lyne met gradiënte m_1 en m_2 is ewewydig as $m_1 = m_2$.
- Twee lyne met gradiënte m_1 en m_2 is loodreg as $m_1 \times m_2 = -1$
- Die gradiënt van 'n horisontale lyn is altyd 0.
- Die gradiënt van 'n vertikale lyn is altyd ongedefinieerd.

- Two lines with gradients m_1 and m_2 are parallel if $m_1 = m_2$.
- Two lines with gradients m_1 and m_2 are perpendicular if $m_1 \times m_2 = -1$
- The gradient of a horizontal line is always 0.
- The gradient of a vertical line is always undefined.

14.3 Die middelpunt tussen twee punte A(x_1;y_1) en B(x_2;y_2) op die Cartesiese vlak het koördinate
The midpoint between two points A(x_1;y_1) and B(x_2;y_2) on the Cartesian plane has co-ordinates

$$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$$

14.4 'n Transformasie gee 'n beeld van die oorspronklike vorm. Die beeld van 'n punt P, is P' na die transformasie.

A transformation results in an image of the original shape. The image of point P, after a transformation, is called P'.

★ Translasie:

vertikale translasie: Reël: (x;y) → (x; y+k)
horisontale translasie: Reël: (x;y) → (x+h; y)
translasie in twee rigtings (vertikaal en horisontaal): Reël: (x;y) → (x+h; y+k)

★ Translation:

vertical translation/shift: Rule: (x;y) → (x; y+k)
horizontal translation/shift: Rule: (x;y) → (x+h; y)
translation/shift in two directions (vertical and horizontal): Rule: (x;y) → (x+h; y+k)

★ Refleksie:

Die reël hang af van die lyn van simmetrie (simmetrie-as):
As ‘n punt P(x;y) in die y-as gereflekteer word, is sy beeld P'(-x;y).
As ‘n punt P(x;y) in die x-as gereflekteer word, is sy beeld P'(x;-y).
As ‘n punt P(x;y) in die lyn y = x gereflekteer word, is sy beeld P'(y;x).
As ‘n punt P(x;y) in die lyn y = -x gereflekteer word, is sy beeld P'(-y;-x).

★ Reflection:

The rule depends on the line of symmetry (mirror line):
If a point P(x;y) is reflected in the y-axis, then its image is P'(-x;y).
If a point P(x;y) is reflected in the x-axis, then its image is P'(x;-y).
If a point P(x;y) is reflected in the line y = x, then its image is P'(y;x).
If a point P(x;y) is reflected in the line y = -x, then its image is P'(-y;-x).

★ Rotasie:

‘n Rotasie is in ‘n antikloksgewyse rigting tensy anders gestel.
Die **rotasiepunt** sowel as die **hoek van rotasie** is nodig om ‘n rotasie te beskryf.
As P(x;y) om die oorsprong geroteer word deur 90°, is die beeld P'(-y;x). Kloksgewyse rotasie om die oorsprong deur 90° kan beskryf word as ‘n rotasie deur -90° gegee deur P’(y; -x).
As (x;y) om die oorsprong deur 180° geroteer word, is die beeld P'(-x;-y).

★ Rotation:

A rotation will be considered as having an anticlockwise direction unless stated otherwise.
The **centre of the rotation** as well as the **angle** of rotation are needed to describe a rotation.
If P(x;y) is rotated about the origin through 90°, the image is P'(-y;x). Clockwise rotation about the origin through 90° would be described as a rotation through -90° gives P’(y; -x).
If P(x;y) is rotated about the origin through 180°, the image is P'(-x;-y).

★ Skalering (vergroting of verkleining):

P(*x*;*y*) word P’(*kx*;*ky*).
Scaling met ‘n faktor *k*:

- As *k* > 1 word die veelhoek vergroot;
- as 0 < *k* < 1 word die veelhoek verklein
- as *k* < -1 word die veelhoek vergroot en geroteer om die oorsprong deur 180°;
- as -1 < *k* < 0 word die veelhoek verklein en geroteer om die orsprong deur 180°.

Die "nuwe" veelhoek is gelykvormig aan die oorspronklike veelhoek.

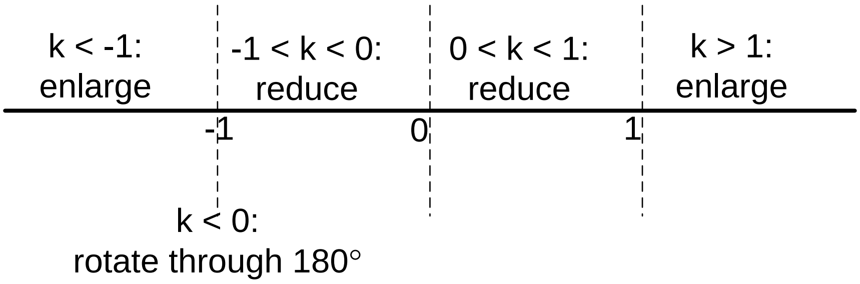
★ Scaling (enlarging or reducing):

P(x;y) becomes P’(kx;ky).
Scaling by a factor k:

- If k > 1 then the polygon will be enlarged (become bigger);
- if 0 < k < 1 the polygon will be reduced (become smaller).
- If k < -1 then the polygon will be enlarged (become bigger) and rotated about the origin through 180°;

- if $-1 < k < 0$ the polygon will be reduced (become smaller) and rotated about the origin through 180°.

The "new" polygon is similar to the original polygon.



Meer as een transformasie kan in dieselfde probleem voorkom. Die volgorde van die transformasies kan 'n invloed hê op die resultaat.

You can have a mixture of transformations in the same problem. The order of the transformations may have an influence on the result.

15. *Skuinssy* / *Hypotenuse*

As c die langste sy van die driehoek is, en as $c^2 > a^2 + b^2$, dan het ons ‘n stomphoekige driehoek.
As c die langste sy van die driehoek is, en as $c^2 < a^2 + b^2$, dan het ons ‘n skerphoekige driehoek.

If c is the longest side of the triangle, and if $c^2 > a^2 + b^2$, then we have an obtuse-angled triangle.
If c is the longest side of the triangle, and if $c^2 < a^2 + b^2$, then we have an acute-angled triangle.

Einde van Algemeen 3 / End of General 3