

ALGEMEEN (5) Ken so goed as moontlik vir deelname aan kompetisies.
 GENERAL (5) Know as well as possible for participation in competitions.

ALGEMEEN 5 / GENERAL 5

Gr 7 - 9

1. Eenhede / Units

Hierdie simbole kry nooit 'n punt aan die einde nie, tensy dit aan die einde van 'n sin staan. These symbols never get a period at the end, except when they appear at the end of a sentence.

<https://physics.nist.gov/cuu/Units/units.html>

<https://physics.nist.gov/cuu/Units/prefixes.html>

Vyf van die SI basiseenhede Five of the SI base units		
Basishoeveelheid Base quantity	Naam Name	Simbool Symbol
lengte / length	meter metre	m
massa / mass	kilogram	kg
tyd / time	sekonde second	s
elektriese stroom electric current	ampere	A

oppervlakte area	vierkante meter square metre	m^2
volume	kubieke meter cubic metre	m^3
spoed, snelheid speed, velocity	meter per sekonde metre per second	m/s
versnelling acceleration	meter per sekonde kwadraat metre per second squared	m/s^2

★ 1 m = 100 cm 1 cm = 0,01 m 1 cm = 10 mm 1 mm = 0,1 cm
 ★ 1 m = 1000 mm 1 mm = 0,001 m 1 km = 1000 m 1 m = 0,001 km

★ 1 decametre = 10 m 1 decametre = 1 dam **decametre is dekameter**

★ 1 decimetre = 0,1 m 1 decimetre = 1 dm **decimetre is desimeter**

★ 1 hectometre = 100 m 1 hectometre = 1 hm

★ 1 hectare = 10 000 m² 1 hectare = 1 ha **hectare is hektaar**

★ 1 hectare = 2,471 acres

★ kilo → duisend / thousand milli → duisendste / thousandth mega → miljoen / million

★ 1 nautical mile = 1852 m

★ 1 (international) knot = 1 nautical mile per hour = 1.852 km per hour

★ 20 knots = 37 km/h

★ 1 mile = 1.609 344 km

★ 1 inch = 2.54 cm 1 foot = 12 inches

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- ★ 1 seemyl = 1852 m
- ★ 1 (internasionale) knoop = 1 seemyl per uur = 1.852 km per uur
- ★ 20 knope = 37 km/h
- ★ 1 myl = 1.609 344 km
- ★ 1 duim = 2.54 cm 1 voet = 12 duim

2. **Spood, tyd, afstand / Speed, time, distance**

$$\text{afstand} = \text{spood} \times \text{tyd} \qquad \text{distance} = \text{speed} \times \text{time}$$

$$\text{tyd} = \frac{\text{afstand}}{\text{spood}} \qquad \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{spood} = \frac{\text{afstand}}{\text{tyd}} \qquad \text{speed} = \frac{\text{distance}}{\text{time}}$$

★ John runs 100 metres in 12.5 seconds. What is his speed in km/h?

Jan hardloop 100 meter in 12.5 sekondes. Wat is sy spood in km/h?

$$\bullet\bullet \text{ speed} = \frac{100 \text{ m}}{12.5 \text{ s}} = \frac{0.1 \text{ km}}{\frac{12.5}{60 \times 60} \text{ h}} = 28.8 \text{ km/h}$$

★ Jan ry 50 km van A na B teen 60 km/h en nog 50 km van B na C teen 90 km/h. Wat is sy gemiddelde spood van A tot C?

Jan drives 50 km at 60 km/h from A to B and another 50 km at 90 km/h from B to C. What is his average speed from A to C?

$$\bullet\bullet \text{ Gemiddelde spood} = \frac{\text{totale afstand}}{\text{totale tyd}}.$$

Totale afstand = 100 km.

$$\text{Totale tyd} = \frac{50 \text{ km}}{60 \text{ km/h}} + \frac{50 \text{ km}}{90 \text{ km/h}} = \frac{50}{60} \text{ h} + \frac{50}{90} \text{ h} = \frac{150+100}{180} \text{ h} = \frac{250}{180} \text{ h} = \frac{25}{18} \text{ h}$$

$$\text{Gemiddelde spood} = \frac{100 \text{ km}}{\frac{25}{18} \text{ h}} = \frac{100 \times 18}{25} \frac{\text{km}}{\text{h}} = 4 \times 18 \frac{\text{km}}{\text{h}} = 72 \text{ km/h}.$$

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}.$$

Total distance = 100 km.

$$\text{Total time} = \frac{50 \text{ km}}{60 \text{ km/h}} + \frac{50 \text{ km}}{90 \text{ km/h}} = \frac{50}{60} \text{ h} + \frac{50}{90} \text{ h} = \frac{150+100}{180} \text{ h} = \frac{250}{180} \text{ h} = \frac{25}{18} \text{ h}$$

$$\text{Average speed} = \frac{100 \text{ km}}{\frac{25}{18} \text{ h}} = \frac{100 \times 18}{25} \frac{\text{km}}{\text{h}} = 4 \times 18 \frac{\text{km}}{\text{h}} = 72 \text{ km/h}.$$

3. **Let op: / Note:**

$$3^2 = 9 \qquad 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \qquad 100 \text{ km/h} = 100 \text{ km} \cdot \text{h}^{-1}$$

$$5^{-1} = \frac{1}{5} \qquad 5^{-\frac{1}{2}} = \frac{1}{5^{\frac{1}{2}}} = \frac{1}{\sqrt{5}} \qquad 7^{\frac{1}{3}} = \sqrt[3]{7}$$

$$12^{\frac{2}{3}} = \sqrt[3]{12^2} \qquad 2^5 \times 2^3 = 2^{5+3} = 2^8 \qquad (2 \times 3)^7 = 2^7 \times 3^7$$

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- As/If $x^2 = 9$ dan/then $x = \sqrt{9}$ of/or $x = -\sqrt{9}$. Dus/Therefore $x = \pm 3$.
- Maar/But $\sqrt{9} = 3$ en/and $-\sqrt{9} = -3$.

$\sqrt{x^2} = |x|$ waar $|x|$ die absolute waarde is van x .

Voorbeelde: $|-7| = 7$, $|-1.0005| = 1.0005$, $|5| = 5$, $|0| = 0$

Voorbeelde / Examples:

$$\sqrt{(-3)^2} = \sqrt{9} = 3 \text{ of } \sqrt{(-3)^2} = |-3| = 3$$

$$\sqrt{-3} \times \sqrt{-3} = (\sqrt{-3})^2 = -3 \quad ; \quad \sqrt{-3} \times \sqrt{-3} \neq \sqrt{(-3)^2}$$

$$\sqrt{-3} \times \sqrt{-3} = \sqrt{(-1) \times 3} \times \sqrt{(-1) \times 3} = \sqrt{3}i \times \sqrt{3}i = -3$$

$$\sqrt{(-1)^2} \neq (\sqrt{-1})^2$$

$$(\sqrt{-1})(\sqrt{-1}) = (\sqrt{-1})^2$$

3. 'n Veelhoek is **reëlmatig** as al sy sye gelyk is en al die binnehoeke ewe groot is. Al die buitehoeke is dus ook ewe groot.

A polygon is **regular** if all its sides are equal and all the interior angles have the same size. All the exterior angles are also equal.

4. **Deelbaarheidsreëls** / **Divisibility rules**

2: ewe getal / even number;

3: die som van syfers deelbaar deur 3 / the sum of digits divisible by 3 (12, 36, ...);

4: die laaste twee syfers deelbaar deur 4/the last two digits divisible by 4 (100, 188, ...);

5: die laaste syfer is 'n 0 of 'n 5 / the last digit is a 0 or a 5;

6: ewe getalle wat ook deelbaar is deur 3 / even numbers that are also divisible by 3;

7: Voorbeelde / Examples:

310 <u>1</u>	31984 <u>4</u>	392
$2 \times \underline{1} = \underline{2}$	$31984 - 2 \times \underline{4} = 31976$	$39 - 2 \times 2 = 35$
$310 - 2 = 308$	$3197 - 2 \times \underline{6} = 3185$	$3 - 2 \times 5 = -7$
$2 \times \underline{8} = 16$	$318 - 2 \times \underline{5} = 308$	
$30 - 16 = 14$	$30 - 2 \times \underline{8} = 14$	

- 8: die laaste 3 syfers deelbaar deur 8 / the last 3 digits divisible by 8;
 die getal moet dus drie keer agtereenvolgens deur 2 gedeel kan word;
 the number must be divisible by 2 three times in succession;
 bv. / e.g. 4552, 523768

- 9: som van die syfers moet deelbaar wees deur 9 / sum of the digits must be divisible by 9;

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Voorbeelde / Examples: 81, 999, 504

Uitgooi van neges:

- a. Is 256618 deelbaar deur 9?
 $2 + 5 + 6 + 6 + 1 + 8 = 28$; $2 + 8 = 10$; $1 + 0 = 1$ wat nie deelbaar is deur 9 nie.
 $\therefore$ 256618 is nie deelbaar deur 9 nie.
- b. Is 356418 deelbaar deur 9?
 $3 + 5 + 6 + 4 + 1 + 8 = 27$; $2 + 7 = 9$ wat deelbaar is deur 9.
 $\therefore$ 356418 is deelbaar deur 9.
- c. Is 3564927 deelbaar deur 9?
 $3 + 5 + 6 + 4 + 9 + 2 + 7 = 36$; $3 + 6 = 9$ wat deelbaar is deur 9.
 $\therefore$ 3564927 is deelbaar deur 9.

In plaas daarvan om die syfers bymekaar te tel, kan die neges en die syfers waarvan die som 9 is, uitgegooi word:

3564927 sal dan só lyk: 356427 $\rightarrow$ 3564 $\rightarrow$ 54 en $5 + 4 = 9$
17831 $\rightarrow$ 731 en $7 + 3 + 1 = 11$
123456789 $\rightarrow$ 12345678 $\rightarrow$ 234567 $\rightarrow$ 3456 $\rightarrow$ 45 $\rightarrow$ 9
126414 $\rightarrow$ 414 $\rightarrow$ 9

Casting out nines:

- a. Is 256618 divisible by 9?
 $2 + 5 + 6 + 6 + 1 + 8 = 28$; $2 + 8 = 10$; $1 + 0 = 1$ which is not divisible by 9.
 $\therefore$ 256618 is not divisible by 9.
- b. Is 356418 divisible by 9?
 $3 + 5 + 6 + 4 + 1 + 8 = 27$; $2 + 7 = 9$ which is divisible by 9.
 $\therefore$ 356418 is divisible by 9.
- c. Is 3564927 divisible by 9?
 $3 + 5 + 6 + 4 + 9 + 2 + 7 = 36$; $3 + 6 = 9$ which is divisible by 9.
 $\therefore$ 3564927 is divisible by 9.

In stead of adding the digits, the nines and the digits of which the sums are 9, can be cast out:

3564927 will then look like this: 356427 $\rightarrow$ 3564 $\rightarrow$ 54 and $5 + 4 = 9$
17831 $\rightarrow$ 731 and $7 + 3 + 1 = 11$
123456789 $\rightarrow$ 12345678 $\rightarrow$ 234567 $\rightarrow$ 3456 $\rightarrow$ 45 $\rightarrow$ 9
126414 $\rightarrow$ 414 $\rightarrow$ 9

<https://www.youtube.com/watch?v=FlndliQa20o> 8:03

- 10: die laaste syfer is 'n 0 / the last digit is a 0;
- 11: tel elke tweede syfer bymekaar en trek af van die som van die orige syfers /
add every second digit and subtract from the sum of the remaining digits
70235: $7 - 0 + 2 - 3 + 5 = 11$ en 11 is deelbaar deur 11;
121: $1 - 2 + 1 = 0$ which is divisible by 11;
2904: $2 - 9 + 0 - 4 = -11$ wat deelbaar is deur 11;
- 12: dit moet deelbaar wees deur 3 **en** deur 4 / it must be divisible by 3 **and** by 4

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5. **Verhouding en eweredigheid:**

Verdeel 24 lekkers in die verhouding 1 : 3.

$$1 + 3 = 4$$

Een hupie het $\frac{1}{4} \times 24 = 6$ en die ander $\frac{3}{4} \times 24 = 18$ of $24 - 6 = 18$.

Ratio and proportion:

Divide 24 sweets in the ratio 1 : 3

$$1 + 3 = 4$$

One pile has $\frac{1}{4} \times 24 = 6$ and the other $\frac{3}{4} \times 24 = 18$ or $24 - 6 = 18$.

Dis goed om die volgende te ken: / It is good to know the following:

As jy elke keer die LK vereenvoudig, dan gee dit die antwoord aan die RK.

If you simplify the answer on the LHS you get the answer on the RHS in each case.

$a : b = c : d$	$12 : 3 = 4 : 1$		
$\frac{a}{b} = \frac{c}{d}$	$\frac{12}{3} = \frac{4}{1}$	$\frac{a}{a+b} = \frac{c}{c+d}$	$\frac{12}{12+3} = \frac{4}{4+1}$
$\frac{a}{c} = \frac{b}{d}$	$\frac{12}{4} = \frac{3}{1}$	$\frac{a-b}{b} = \frac{c-d}{d}$	$\frac{12-3}{3} = \frac{4-1}{1}$
$\frac{b}{a} = \frac{d}{c}$	$\frac{3}{12} = \frac{1}{4}$	$\frac{a}{a-b} = \frac{c}{c-d}$	$\frac{12}{12-3} = \frac{4}{4-1}$
$\frac{c}{a} = \frac{d}{b}$	$\frac{4}{12} = \frac{1}{3}$	$\frac{a+b}{a-b} = \frac{c+d}{c-d}$	$\frac{12+3}{12-3} = \frac{4+1}{4-1}$
$\frac{a+b}{b} = \frac{c+d}{d}$	$\frac{12+3}{3} = \frac{4+1}{1}$		

Die verhouding $4 : 6 = 2 : 3$ en die verhouding $6 : 9 = 2 : 3$. Die twee verhoudings is dus dieselfde as hulle vereenvoudig word en is dus **ekwivalent**.

The ratio $4 : 6 = 2 : 3$ and the ratio $6 : 9 = 2 : 3$. The two ratios are the same after simplification and are therefore **equivalent**.

Die verhouding $123 : 321$ vereenvoudig na $41 : 107$. Dis dieselfde as $\frac{123}{321} = \frac{41}{107}$.

The ratio $123 : 321$ simplifies to $41 : 107$. It is the same as $\frac{123}{321} = \frac{41}{107}$.

★ Die verhouding van Annie se salaris tot Bennie s'n is $\frac{5}{4}$. Die verhouding van Bennie se salaris tot Callie s'n is $10 : 9$. Hoeveel word Callie betaal as Annie R7500 kry?

$$A : B = 5 : 4$$

$$B : C = 10 : 9$$

$A : B : C = 25 : 20 : 18$kry die KGV(4, 10).

Annie se salaris gedeel deur 25: $7500 \div 25 = 300$

Dit is dan 25 dele van R300 elk. Callie kry 18 sulke dele van R300 elk:

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$18 \times 300 = 5400$
 Callie verdien R5400.

★ Ann beats Bea by 20 m in a 100 m race. Bea beats Cate by 20 m in a 100 m race. By what distance does Ann beat Cate?

In the same time that Ann runs a m, Bea runs b m and Cate runs c m.

$$\begin{aligned} a : b &= 100 : 80 \\ b : c &= 100 : 80 \end{aligned}$$

$$\begin{aligned} a : b : c &= 500 : 400 : 320 \dots \dots \dots \text{LCM}(80, 100) \\ &= 100 : 80 : 64 \end{aligned}$$

If Ann runs 100 m then Cate runs 64 m; so Ann beats Cate by 36 m.

6. Long division in algebra / Langdeling in algebra:

$$\begin{array}{r} x+2 \\ x^2+1 \overline{) x^3+2x^2-3x+2} \\ \underline{x^3 + x} \\ 2x^2-4x \\ \underline{2x^2 + 2} \\ -4x \end{array}$$

$$\frac{x^3+2x^2-3x+2}{x^2+1} = x+2 \text{ res/remainder } -4x$$

x^2+1 is dus nie 'n faktor van x^3+2x^2-3x+2 nie. As dit 'n faktor was, sou die res gelyk gewees het aan 0.

x^2+1 is therefore not a factor of x^3+2x^2-3x+2 . If it was a factor, the remainder would be equal to 0.

7. Algebra: Faktore / Factors

★ Laat $f(x) = x^2 - 5x + 6$ Let $f(x) = x^2 - 5x + 6$

As $f(2) = 0$, If $f(2) = 0$,

dan is $(x-2)$ 'n faktor van $f(x)$: then $(x-2)$ is a factor of $f(x)$:

$$f(x) = x^2 - 5x + 6 \text{ en dan is / and then } f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$$

★ $x-a$ is a factor of $x^2+2ax-3$. Find the numerical value(s) of a .
 $x-a$ is 'n faktor van $x^2+2ax-3$. Vind die numeriese waarde(s) van a .

$$f(x) = x^2 + 2ax - 3$$

$$\text{Laat } f(a) = 0: f(a) = a^2 + 2a^2 - 3 = 3a^2 - 3 = 3(a^2 - 1) = 3(a-1)(a+1)$$

$$3(a-1)(a+1) = 0$$

$$a = 1 \text{ of/or } a = -1$$

8. Totale aantal reghoeke (vierkante ingesluit), S , op 'n $n \times m$ -bord word gegee deur

Total number of rectangles (squares included), S , on an $n \times m$ board is given by

$$S = \left(\sum_{i=1}^m i \right) \times \left(\sum_{j=1}^n j \right) = \frac{m(m+1)}{2} \times \frac{n(n+1)}{2} \dots \dots n \times m \text{ bord/board } \textcircled{1}$$

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$$S = \left(\frac{1}{2} n(n+1) \right)^2 = \left(\frac{n}{2} (n+1) \right)^2 \dots\dots n \times n \text{ bord/board } (m = n) \text{ ②}$$

② gee die aantal vierkante plus die aantal reghoeke vir 'n vierkantige bord. Vir $n = 8$ is dit 1296.

② gives the number of squares plus the number of rectangles for a square board. For $n = 8$ it is 1296.

Voorbeeld/Example:

+	+	+	+				
+	+	+	+				
					+	+	
					+	+	
					+	+	
					+	+	

Totale aantal vierkante, S , op 'n $n \times n$ -bord word gegee deur

Total number of squares, S , on an $n \times n$ board is given by

$$S = \sum_{k=1}^n k^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$n = 8: S = 204$$

Voorbeeld/Example:

+	+						
+	+						
			+	+			
			+	+	+		
				+	+		

<http://puzzles.nigelcoldwell.co.uk/twentyseven.htm>

<https://phoxis.org/2010/04/16/rectangles-in-checked-board/>

<https://www.youtube.com/watch?v=hrDm9Epw1Fw>

Jy kan bewyse in boeke en op die Internet vind van hoe die formules afgelei is.

You can find proofs in books and on the Internet of the deduction of the formulas.

9. KGV en GGD / LCM and GCD (HCF)

$$\begin{array}{lcl} 12 & = & 2 \times 2 \times 3 \\ 15 & = & 3 \times 5 \end{array}$$

KGV: jy soek 'n veelvoud en dus soek jy een getal in elke kolom.

GGD: jy soek die grootste getal wat in albei getalle indeel en dus soek jy die getalle wat in al die rye van 'n kolom voorkom.

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$$\text{KGV}_{12,15} = 2 \times 2 \times 3 \times 5$$

$$\text{LCM}_{12,15} = 2 \times 2 \times 3 \times 5$$

$$\text{GGD}_{12,15} = 3$$

$$\text{GCD}_{12,15} = \text{HCF}_{12,15} = 3$$

$$\begin{array}{rcl} 120 & = & 2 \times 2 \times 2 \times 3 \times 5 \\ 45 & = & 3 \times 5 \times 3 \end{array}$$

$$\text{KGV}_{120,45} = 2 \times 2 \times 2 \times 3 \times 5 \times 3$$

$$\text{LCM}_{120,45} = 2 \times 2 \times 2 \times 3 \times 5 \times 3$$

$$\text{GGD}_{120,45} = 3 \times 5$$

$$\text{HCF}_{120,45} = 3 \times 5$$

$$*\text{KGV}_{a,b} \times \text{GGD}_{a,b} = a \times b$$

$$*\text{LCM}_{a,b} \times \text{GCD}_{a,b} = a \times b$$

9. Kort paaie vir **vermenigvuldiging** / Short ways for **multiplication**

- Vermenigvuldiging met **25** is dieselfde as $(\times 100 \div 4)$
 Multiplication by **25** is the same as $(\times 100 \div 4)$
 - $34 \times 25 = 34 \times (100 \div 4) = 34 \times 100 \div 4 = 3400 \div 4 = 850$
- Vermenigvuldiging met **99** is dieselfde as $\times (100 - 1)$
 Multiplication by **99** is the same as $\times (100 - 1)$
 - $34 \times 99 = 34 \times (100 - 1) = 3400 - 34 = 3366$
- Vermenigvuldiging met **125** is dieselfde as $(\times 1000 \div 8)$
 Multiplication by **125** is the same as $(\times 1000 \div 8)$
 - $34 \times 125 = 34 \times (1000 \div 8) = 34 \times 1000 \div 8 = 34000 \div 8 = 4250$
- Let op dat $4250 \div 5 = 850$ en $125 \div 5 = 25$.
 Note that $4250 \div 5 = 850$ and $125 \div 5 = 25$.
- $49 = 50 - 1$
- $51 = 50 + 1$
- Jy kan nog ander kortpaaie self uitdink. / You can figure out more short cuts yourself.